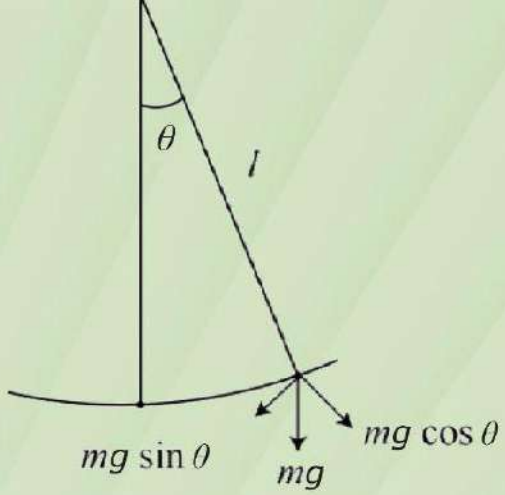
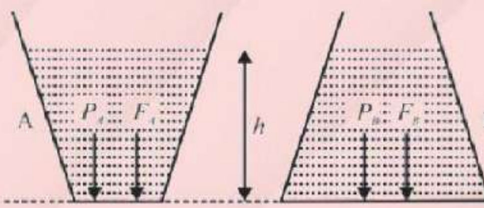




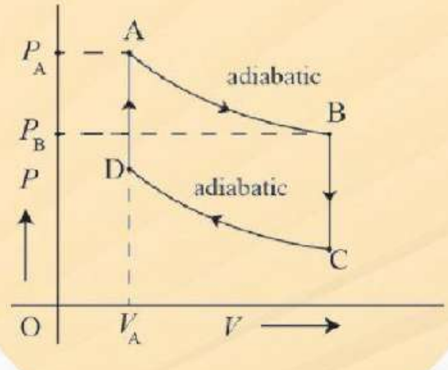
Continuous Professional Development

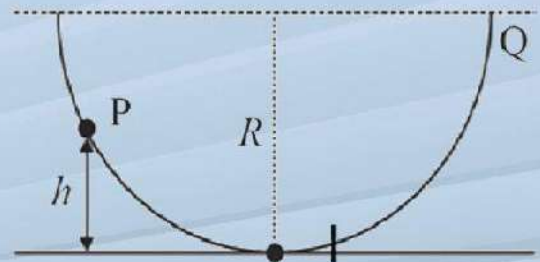
Self-Assessment

Physics Teachers

303 MCQs with Explanation

Class XI





CONTENTS

1. Laws of Motion

Module 1.1 Laws of Motion Part-1

In all questions,it is assumed that

(i) strings are massless and inextensible unless otherwise stated.

(ii) surfaces in contacts are frictionless unless otherwise stated.

Q.1.1.1 Two masses m_1 and m_2 are connected as shown in Fig.Q.1.1.1. The system is in equilibrium. Calculate the tensions, T_1 and T_2. (g = 10 m/s^2)

(A) T_1= 50 N, T_2= 60 N

(B) T_1= 60 N, T_2= 50 N

(C) T_1=110 N, T_2=0 N

(D) T_1=30 N, T_2=25 N

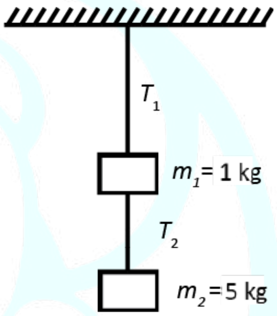


Fig.Q.1.1.1

Q.1.1.2 A person, wearing a seat belt, is driving a car. She applies brakes on seeing a child on the road ahead. She is less likely to fall forward and hit against the steering wheel because the seat belt

(A) increases the time of impact

(B) decreases the time of impact

(C) is of a very high tensile strength

(D) keeps driver fixed to the seat

Q.1.1.3 Two friends, Ram and Sham are traveling in two different buses A and B. Bus A is moving with a constant velocity while bus B is moving with a constant acceleration. Ram and Sham, while remaining seated in their fixed positions in the buses, toss coins vertically upwards. If they are agile enough to be able to catch the coins if they fall on their hands, then which among the following statements is correct?

(A) Both Ram and Sham are able to catch the respective coins.

(B) Ram catches the coin while Sham will not be able to catch the coin.

(C) Sham catches the coin, but Ram will not be able to catch the coin.

(D) Neither Ram, nor Sham will be able to catch the coins

Q.1.1.4 A block of mass 13 kg is resting on a horizontal surface (Fig.Q.1.1.2). A force of 20 N is applied as shown in the figure. (i) What is the horizontal force required to keep the block in equilibrium? (ii) What is the normal reaction? (g=10 m/s^2, cos37^0 = ⅘, sin37^0 = ⅗)

(A) (i) 16 N (ii) 142 N

(B) (i) 130 N (ii) $20\sqrt{2}$ N

(C) (i) 12 N (ii) $10\sqrt{2}$ N

(D) (i) 20 N (ii) 130 N

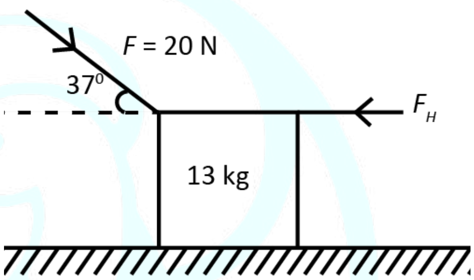


Fig. Q.1.1.2

Q.1.1.5 The system shown in Fig.Q.1.1.3 is in equilibrium. Calculate tensions T_1 and T_2.

(A) $T_1 = 133.3$ N, $T_2 = 166.7$ N

(B) $T_1 = 166.7$ N, $T_2 = 133.3$ N

(C) $T_1 = 100$ N, $T_2 = 50$ N

(D) $T_1 = 200$ N, $T_2 = 166$ N

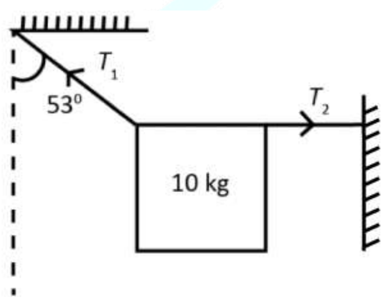


Fig.Q.1.1.3

Q.1.1.6 The system shown in Fig.Q.1.1.4 is in equilibrium. The tension T_1 and T_2 are

(A) $25\ \text{N},\ 25\sqrt{3}\ \text{N}$

(B) $50\ \text{N},\ 25\ \text{N}$

(C) $25\sqrt{3}\ \text{N},\ 50\sqrt{3}\ \text{N}$

(D) $25\ \text{N},\ 50\sqrt{3}\ \text{N}$

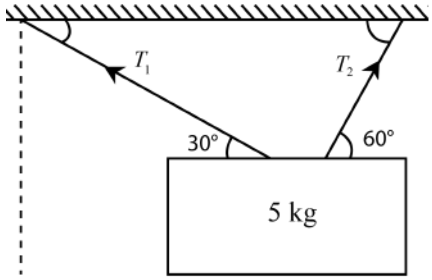


Fig.Q.1.1.4

Q.1.1.7 The system shown in Fig.Q.1.1.5 is in equilibrium.The tension T in the string and the normal reaction R on the mass are

(A) $T = 20\ \text{N}$, $R = 20\sqrt{3}\ \text{N}$

(B) $T = 20\sqrt{3}\ \text{N}$, $R = 20\ \text{N}$

(C) $T = 10\ \text{N}$, $R = 10\sqrt{3}\ \text{N}$

(D) $T = 10\sqrt{3}\ \text{N}$, $R = 10\ \text{N}$

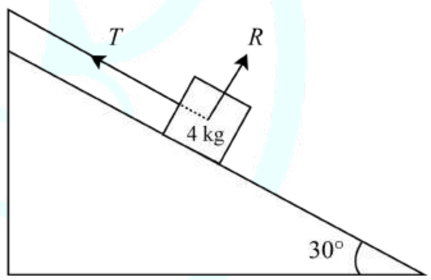


Fig.Q.1.1.5

Q.1.1.8 A uranium nucleus ${}_{92}U^{238}$ decays emitting an α-particle (${}_{2}He^{4}$),

$${}_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}He^{4} + 4.29\ \text{MeV}$$

The energy of α-particle expressed as a percentage of total energy released is

(A) 2 %

(B) 50 %

(C) 98 %

(D) 100 %

Q.1.1.9 Fig.Q.1.1.6 shows two blocks M and m in equilibrium on an inclined plane. M is

(A) 2 kg

(B) 10 kg

(C) 5 kg

(D) 10/3 kg

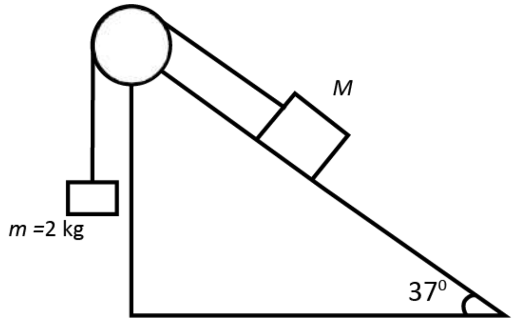


Fig.Q.1.1.6

Q.1.1.10 A load is raised by a rope, from a rest position to another at a height h meter above. The greatest tension the rope can bear is ten times the load. What is the least time in which this ascent can take place? (Take $g = 10$ m/s^2)

(A) $\sqrt{\frac{h}{5}}$

(B) $\sqrt{\frac{h}{2}}$

(C) $\frac{\sqrt{h}}{2}$

(D) $\sqrt{\frac{2h}{9}}$

Module 1.2 Laws of Motion Part-2

Q. 1.2.1 The velocity of a body of mass 2 kg as a function of t is given by $v = 2t\,\hat{i} + t^2\,\hat{j}$. Find the momentum and the force acting on it at time $t = 2$ s.

(A) $8\,(\hat{i} + \hat{j})$ kg m/s, $(4\,\hat{i} + 8\,\hat{j})$ N

(B) 11.31 kg m/s, 8.94 N

(C) $8\,(\hat{i} + \hat{j})$ kg m/s, $8\hat{j}$

(D) $8\,\hat{i}$ kg m/s, $(4\,\hat{i} + 8\,\hat{j})$ N

Q.1.2.2 A block of mass 4 kg is placed on a frictionless plane surface and a force of 10 N is applied at an angle of 60° with the vertical as shown in Fig.Q.1.2.1 Calculate the acceleration of the block and the normal reaction acting on it ($g = 10$ m/s^2).

(A) $a = \frac{5}{4}$ m/s^2, $N = 40$ N

(B) $a = 4$ m/s^2, $N = \frac{5}{4}$ N

(C) $a = \frac{5}{4}\sqrt{3}$ m/s^2, $N = 45$ N

(D) $a = \frac{5}{4}$ m/s^2, $N = 45$ N

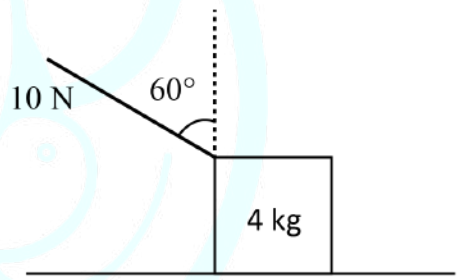


Fig.Q.1.2.1

Q.1.2.3 Two blocks of mass 5 kg and 10 kg are placed in contact as shown in Fig.Q.1.2.2. A force of 10 N is applied on the 5 kg block. Calculate the acceleration of the 10 kg block and its normal reaction on the 5 kg block. Assume that there is no friction between the blocks and between the blocks and the surface on which they are resting.]

(A) $N = \frac{20}{3}$ N, $a = 1$ m/s^2

(B) $N = 5$ N, $a = 2$ m/s^2

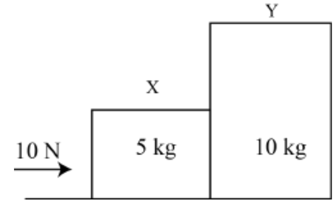


Fig.Q.1.2.2

(C) $N = \frac{20}{3}$ N, $a = \frac{2}{3}$ m/s^2

(D) N =20 N, $a = \frac{1}{2}$ m/s^2

Q.1.2.4 A cricket ball of mass 160 g moving at a speed of 120 km/hr is deflected through an angle of 60° such that there is no change in the magnitude of the velocity of the ball. Calculate the impulse (in kg m/s) imparted to the ball by the bat.

(A) 0

(B) $\frac{16\sqrt{3}}{3}$ in a direction making an angle 90 degrees with the original direction

(C) $\frac{8\sqrt{3}}{3}$ in a direction making an angle 150 degrees with the original direction

(D) $\frac{16}{3}$ in a direction making an angle 120 degrees with the original direction

Q.1.2.5 Three blocks are resting on a frictionless table, while a force 20 N is applied on the system as shown in the Fig.Q.1.2.3. Calculate the magnitudes of the acceleration and the normal reaction by block A on B, and block C on B.

(A) a = 4 m/s^2 N_1 = 5 N, N_2 = 10 N

(B) a = 2/3 m/s^2 N_1= 50/3 N, N_2= 10 N

(C) a = 2 m/s^2 N_1 = 10 N, N_2 = 5 N

(D) a = 2/3 m/s^2 N_1=5 N, N_2 = 50/3 N

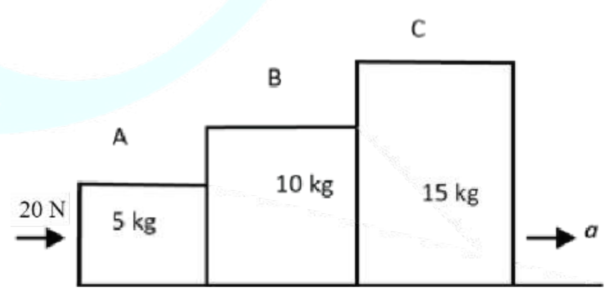


Fig.Q.1.2.3

Q.1.2.6 Blocks of mass 10 kg and 20 kg are at rest on a smooth frictionless table. They are tied with strings and pulled as shown in Fig. Q.1.2.4 , where T_1 =10 N. Find the acceleration experienced by the system and the tension T_2 .

(A) $a = 1$ m/s^2, $T_2 = 1$ N

(B) $a = 1/3$ m/s^2, $T_2 = 5$ N

(C) $a = 1$ m/s^2, $T_2 = 10$ N

(D) $a = 1/3$ m/s^2, $T_2 = 20/3$ N

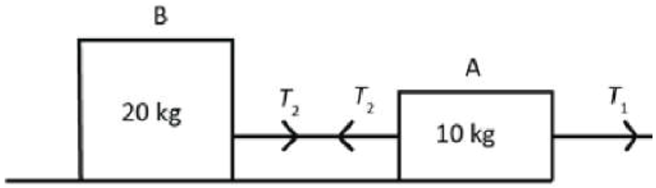


Fig.Q.1.2.4

Q.1.2.7 Two unequal masses m_1 and m_2 are attached by means of a string of length l as shown in Fig.Q.1.2.5. The pulley of radius r is massless and frictionless. The accelerations of the two wings will be related as

(A) $a_1 = - a_2$

(B) $a_1 = a_2$

(C) $a_1 = \frac{m_2}{m_1} a_2$

(D) $a_1 = - \frac{m_2}{m_1} a_2$

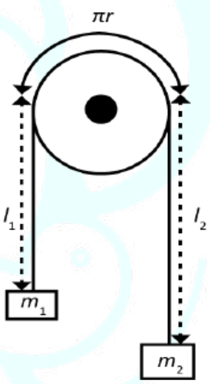


Fig.Q.1.2.5

Q.1.2.8 A bullet of mass m with a velocity v is just able to penetrate a block of width d. If the block offers a uniform resistance R, then v will be given by

(A) $\sqrt{\frac{2Rd}{m}}$

(B) $\sqrt{\frac{2Rdg}{mg+R}}$

(C) $\sqrt{\frac{Rd}{2m}}$

(D) $\sqrt{\frac{Rdg}{mg+R}}$

Q.1.2.9 A thin plate can just withstand a weight of 12.5 kg. It is being raised vertically upward with increasing acceleration, with a mass placed on it. What is the value of the mass (in kg correct up to one decimal place) if the plate breaks when the acceleration is 1.25 m/s^2 ? (Take g = 10 m/s^2)

(A) 12.5
(B) 11.5
(C) 11.1
(D) 10.8

Q.1.2.10 A body of mass m is moving vertically downward for a time t second from rest. it is brought to rest after traveling another vertical length l. What is the value of the vertically upward force applied if there is no air resistance ?

(A) $mg\,(1 + \frac{gt^2}{2l})$

(B) $mg\,(1 - \frac{gt^2}{2l})$

(C) $mg\,(1 + \frac{gt^2}{l})$

(D) $mg\,(1 - \frac{gt^2}{l})$

Module 1.3 Laws of Motion Part-3

In the following questions, take $g = 10$ m/s^2, μ_s is the coefficient of static friction and μ_k is the coefficient of kinetic friction.

Q.1.3.1 An object of mass 5 kg is placed on an inclined plane with angle of inclination as 53°. What is the force that is trying to pull it down the incline? ($\mu_s = 0.2$, $\mu_k = 0.1$)

- **(A)** 40 N
- **(B)** 8 N
- **(C)** 6 N
- **(D)** 30 N

Q.1.3.2 An object of mass 10 kg is placed on an inclined plane with angle of inclination as 53°. What is the force of limiting friction acting on the object? ($\mu_s = 0.2$, $\mu_k = 0.1$)

- **(A)** 6 N
- **(B)** 12 N
- **(C)** 40 N
- **(D)** 20 N

Q.1.3.3 An object of mass 2.5 kg is placed on an inclined plane with angle of inclination as 53°. Will this object slide down the plane? If yes, what is the force of friction on it and what is its acceleration?
($\mu_s = 0.2$, $\mu_k = 0.1$)

- **(A)** Yes, 3 N, 1.2 m/s^2
- **(B)** No, 3 N, 0 m/s^2
- **(C)** Yes, 1.5 N, 7.4 m/s^2
- **(D)** No, 20 N, 0 m/s^2

Q.1.3.4 An object of mass 5 kg is placed on an inclined plane with angle of inclination as 53°. What force along the plane is required to keep the body at rest? ($\mu_s = 0.2$, $\mu_k = 0.1$)

- **(A)** $F = 34$ N

(B) $F = 37$ N

(C) $F = 40$ N

(D) $F = 6$ N

Q.1.3.5 The force of 62 N applied along the plane on an object of mass 10 kg keeps the object at rest. The angle of inclination of the plane is 53°.The force of limiting friction and coefficient of static friction are

(A) 80 N, 0.225

(B) 62 N, 0.3

(C) 18 N, 0.225

(D) 18 N, 0.3

Q.1.3.6 What force applied along the plane is required to move a body of mass 2.5 kg up the inclined plane with a constant velocity? ($\mu_s = 0.3$, $\mu_k = 0.2$)

(A) 24.5 N

(B) 18 N

(C) 23 N

(D) 46 N

Q.1.3.7 What force applied along the plane is required to move the body of mass 5 kg up to an inclined plane with an acceleration of 1 m/s^2? ($\mu_s = 0.2, \mu_k = 0.1$)

(A) 40 N

(B) 44 N

(C) 48 N

(D) 45 N

Q.1.3.8 What force applied along the plane is required to bring down the body of mass 10 kg on the inclined plane with a constant velocity? ($\mu_s = 0.2, \mu_k = 0.1$)

(A) 74 N

(B) 86 N

(C) 80 N

(D) 6 N

Q.1.3.9 A force F is applied on the body B as shown in Fig.Q.1.3.1.The mass of B is m and A is The coefficient of friction between the floor and body B is 0.1 between bodies B and A is 0.2.

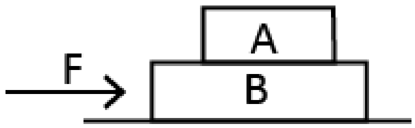


Fig.Q.1.3.1

Which of the following statements is true? The two bodies will move together if the value of F = 0.25 mg

1. The body A will slip with respect to body B if F= 0.5 mg
2. The two bodies will move together if the value of F= 0.5 mg
3. The bodies will be at rest if the value of F= 0.1 mg
4. The two bodies will move together if the value of F= 0.25 mg
5. The maximum value of F for which the two bodies will move together is 0.45 mg

(A) All statements except 3 are false

(B) All statements except 2 are true

(C) Statements 1, 3, 4 are true and 2 and 5 are false

(D) Statements 1, 2, 5 are true and 3 and 4 are false

Q.1.3.10 Three forces F_1, F_2, *and* F_3 acting at a point P on a body. The body moves with a uniform speed.

Which of the following statement/s is/are correct?

(1) The forces are coplanar.

(2) The torque acting on the body at about any point due to these three forces is zero.

(A) Both one and two are True

(B) One is true and two is False

(C) One is False and two is True

(D) Both one and two are False

ANSWERS

Module 1.1 Laws of Motion Part -1

A.1.1.1 (B)

Free Body Diagram for $m_2 = 5$ kg is shown in Fig.A.1.1.1(a)

We have

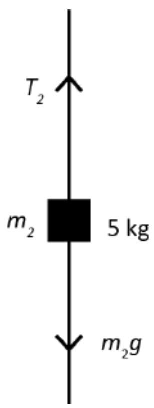


Fig A.1.1.1(a)

$$m_2g - T_2 = m_2a$$

$$5\times10 - T_2 = 5\times0 \; [\because a = 0]$$

$$50 - T_2 = 0 \quad \text{or } T_2 = 50 \text{ N}$$

Fig. A.1.1.1(b) shows free body diagram for $m_1 = 1$ kg. From Newton's law

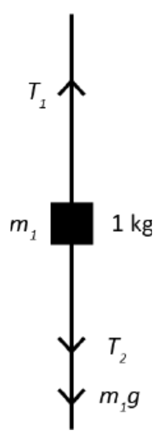


Fig. A.1.1.1(b)

$$m_1g + T_2 - T_1 = m_1a$$

As the body is at rest, $a = 0$.Therefore,

$$10 + 50 - T_1 = 0$$

or , $T_1 = 20$ N

A.1.1.2 (A) True

When the driver suddenly applies brakes, the car comes to rest almost immediately. If she is not wearing the seat belt, the lower part of the body of the driver on the seat also comes to rest with the car but the upper part of the body continues moving with the initial momentum (which was there at the time of application of the brakes) and hence she hits against the steering wheel almost instantly. However, when the driver is wearing a seatbelt, the belt increases the time taken by the body to reach the steering wheel while continuously slowing it down. Hence by the time the body reaches the steering wheel, it has already either come to rest or the impact is negligible. In other words the seat belt facilitates increasing the time of impact,

(C) False. The tensile strength of the seat belt is not a factor that is used here as the seat belt gets pulled out while continuously slowing down the body of the driver.

(D) False. If the seat belt would keep the driver fixed to the seat then the seat belt will cause an impact on the body and there shall be a whiplash on the body due to the seat belt.

A.1.1.3 (B)

There is no relative motion in the horizontal direction between Ram and coin. Therefore , Ram catches the coin. There is relative motion between Sham and coin in a horizontal direction due to acceleration of bus B. Therefore, Sham will not be able to catch the coin.

A.1.1.4 (A) Refer to Figure A.1.1.2

Let F_H be the horizontal force required to keep the body in equilibrium. For equilibrium in a horizontal direction

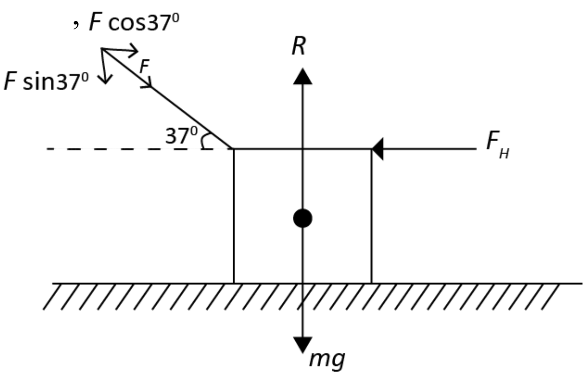


Fig.A.1.1.2

$$[20\cos 37°\hat{i} + (-\vec{F}_H)] = \vec{0}$$

$$F_H = 20\times\frac{4}{1} = 14\text{ N}$$

For equilibrium in a vertical direction,

$$\vec{R} = \hat{j}\,(20\sin 37° + 13\times g)$$

$$\vec{R} = \hat{j}\left(20\times\frac{3}{1} + 130\right)\text{ N}$$

$$= (142\hat{j})\text{ N}$$

A.1.1.5 (B)

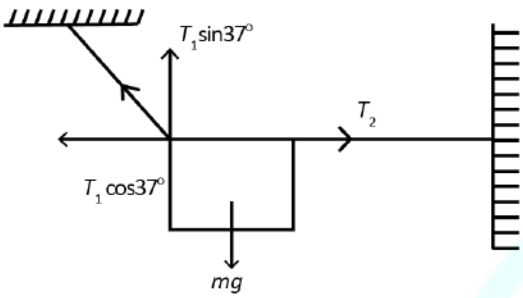


Fig.A.1.1.3

Resolving T_1 into rectangular components (Fig A.1.1.3),

$$T_1 = -T_1\cos 37^{\circ}\,\hat{i} + T_1\sin 37^{\circ}\,\hat{j}$$

As the system is in equilibrium (a = 0)

Along the x-axis,

$$T_2\hat{i} - T_1\cos 37^{\circ}\,\hat{i} = \vec{0}$$

$$T_2\hat{i} = T_1\cos 37^{\circ}\,\hat{i} = T_1\frac{4}{5}\hat{i}$$

Along y- axis

$$T_1\sin 37^{\circ}\,\hat{j} = mg$$

$$T_1\frac{3}{5} = 10\times10 \qquad (g = 10\ \mathrm{m/s^2})$$

$$T_1 = \frac{500}{3}\ \mathrm{N} = 146.7\ \mathrm{N}$$

$$T_2 = T_1\frac{4}{5} = \frac{500}{3}\times0.8 = 133.3\ \mathrm{N}$$

A.1.1.6 (A)

Fig.A.1.1.4 shows the force diagram.

Resolving, T_1 and T_2 into rectangular components as shown, for equilibrium we get,

Along x axis,

$$T_1\cos30^{0} = T_2\cos 60^{0} \qquad (1.1.6.1)$$

$$T_1\frac{\sqrt{3}}{2} = T_2\frac{1}{2}$$

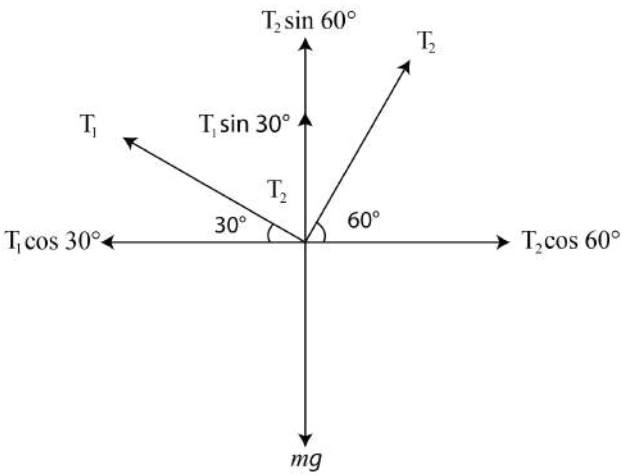


Fig.A.1.1.4

Along y-axis

$$T_2 \sin 60^0 + T_1 \sin 30^0 = mg \qquad (1.1.6.2)$$

$$T_2 \frac{\sqrt{3}}{2} + T_1 \frac{1}{2} = 5\times 10 \qquad (1.1.6.3)$$

Therefore $T_1 + \sqrt{3}\, T_2 = 100$.

So, from Eq. (1.1.6.2),

$4\, T_1 = 100$, $T_1 = 25$ N, and $T_2 = 25\sqrt{3}$ N

A.1.1.7 (A)

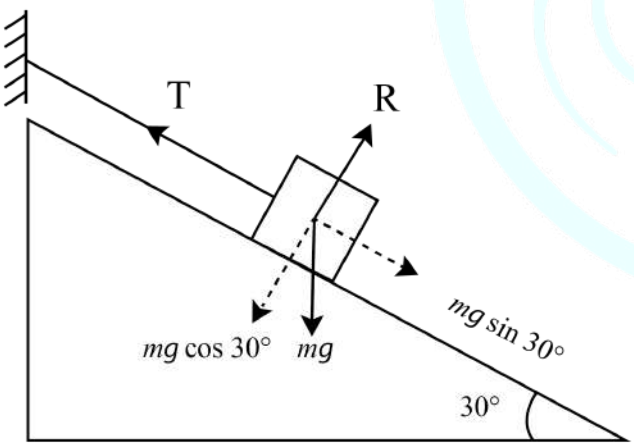


Fig.A.1.1.5

Fig. A.1.1.5 shows the force diagram.

For equilibrium

(i) Along the plane

$$mg \sin 30^o - T = 0$$

$$T = 4\times 10\times \frac{1}{2} = 20 \text{ N}$$

(ii) Perpendicular to plane

$$R = mg \cos 30^o, \quad R = 4\times 10\times \frac{\sqrt{3}}{2} = 20\sqrt{3} \text{ N}$$

A.1.1.8 (C)

Given

${}_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}He^{4}$ (α-particle) + 4.29 MeV

Initially, ${}_{92}U^{238}$ is at rest. Initial momentum is zero.

From the law of conservation of momentum, after splitting, the total momentum of the system (α-particle + ${}_{90}Th^{23}$) must be equal to zero. Therefore,

$$m_{\alpha} v_{\alpha} + m_{Th} v_{Th} = 0$$

$$\text{Or, Magnitude of } v_{Th} = \frac{m_{\alpha} v_{\alpha}}{m_{Th}} \qquad (1.1.8.1)$$

From the law of conservation of energy,

KE of α-particle + KE of Th = 4.29 MeV

$$\frac{1}{2} m_{\alpha} v_{\alpha}^{2} + \frac{1}{2} m_{Th} \left(\frac{m_{\alpha} v_{\alpha}}{m_{Th}} \right)^{2} = 4.29 \text{ MeV}$$

$$\frac{1}{2} m_{\alpha} v_{\alpha}^{2} \left(1 + \frac{m_{\alpha}}{m_{Th}} \right) = 4.29 \text{ MeV}$$

$$\frac{1}{2} m_{\alpha} v_{\alpha}^{2} \left(\frac{m_{Th} + m_{\alpha}}{m_{Th}} \right) = 4.29 \text{ MeV}$$

$$\frac{1}{2} m_{\alpha} v_{\alpha}^{2} \left(\frac{238}{234} \right) = 4.29 \text{ MeV}$$

$$\frac{1}{2} m_{\alpha} v_{\alpha}^{2} = 4.29 \left(\frac{234}{238} \right) \text{ MeV}$$

= 98% of 4.29 MeV

A.1.1.9 (D) Fig. A.1.1.6 shows the force diagram. For equilibrium of mass M

$$Mg \sin 37^{o} = T \qquad (1.1.9.1)$$

For equilibrium of mass m;

$$T = mg \qquad (1.1.9.2)$$

From Eq. (1.1.9.1) and Eq. (1.1.9.2)

$$Mg\sin 37^0 = mg$$

$$\therefore M = 2 \times \frac{5}{3} = \frac{10}{3}\ \text{kg}$$

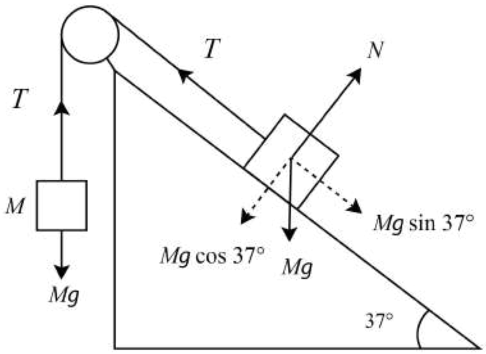


Fig A.1.1.6

A.1.1.10 (D)

Refer to Fig A.1.1.7(a).

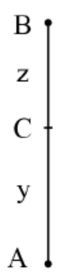


Fig. A.1.1.7(a)

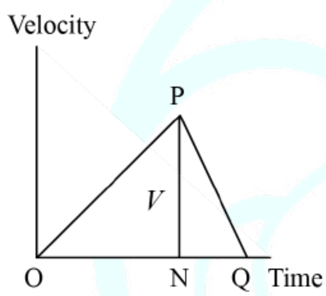


Fig. A.1.1.7(b)

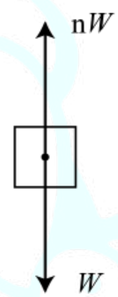


Fig. A.1.1.7(c)

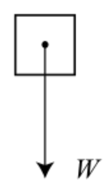


Fig. A.1.1.7(d)

A is the initial position of rest from where the load is lifted, and it is taken up to B, which is its final position of rest. C is an intermediate point upto which it accelerates due to the net unbalanced force, resulting out of the tension in the rope and the load, and thereafter it decelerates.

Let, AC and CB be equal to y and z respectively.

$$\text{So, } h = y + z \qquad (1.1.10.1)$$

Now, we shall relate h with time. For that we shall draw the velocity versus time graph. The load starts from rest, accelerates uniformly, attains a highest velocity V, and then decelerates and finally comes to rest. So, the graph will be like what has been shown at Fig.A.1.1.7(b), as we know that the velocity versus time graph of a particle moving under uniform acceleration is a straight line.

The total displacement is equal to the area under the triangle OPQ , which is equal to $\frac{1}{2}$ OQ× PN= $\frac{1}{2}Vt = h$.

$$\text{So, } t = \frac{2h}{V} \tag{1.1.10.2}$$

For the time being least, value of V must be the highest possible, and this happens when the tension in the rope due to the pull is highest (= $n\ W$, where W is the load and $n = 10$).

Correspondingly, we have the FBDs for the parts AC and CB; which are Fig A.1.1.8 (c) and (d) respectively.
We have

$$nW - W = \frac{Wf}{g}; \text{ where } f \text{ is the acceleration}$$

$$f = (n-1)\ g \tag{1.1.10.3}$$

And the deceleration takes place at the rate of acceleration due to gravity. So, the square of the highest velocity can be expressed in two ways, which are as follows.

$$V^2 = 2(n-1)gy \tag{1.1.10.4}$$

$$\text{And also, } V^2 = 2gz \tag{1.1.10.5}$$

So, from Eqns. (1.1.10.1; 10.4 and 10.5), we get,

$$h = \frac{V^2}{2g}\left(1 + \frac{1}{n-1}\right) = \frac{V^2}{2g}\left(\frac{n}{n-1}\right) \tag{1.1.10.6}$$

Hence, from Eqn. (1.1.10.2), we get

$$t = \sqrt{\frac{2n\,h}{(n-1)g}} \tag{1.1.10.7}$$

So, putting $n = 10$, $g = 10$, we get $t = \sqrt{\frac{2h}{9}}$

A.1.2.1(A)

Momentum $\vec{p} = m\vec{v}$

At $t = 2$ s

$$\vec{p} = 2x\,(4\hat{i} + 4\hat{j}) = 8(\hat{i} + \hat{j})$$

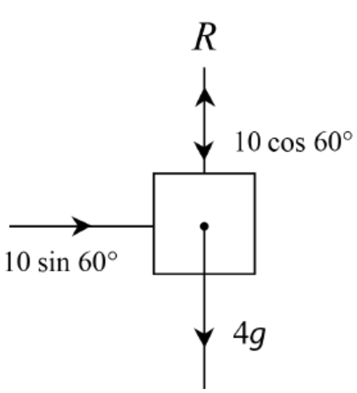
$$\vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\frac{d\left(2t\hat{i} + t^2\hat{j}\right)}{dt}$$

$$\vec{F} = m\,2\hat{i} + m\,2t\hat{j}$$

At $t = 2$ sec,

$$\vec{F} = \left(4\hat{i} + 8\hat{j}\right)\text{ N}$$

A.1.2.2 (C) Fig A.1.2.1 shows force diagram.



FigA.1.2.1

Resolving $\vec{F}$, the applied force, we get

$$\vec{F} = \hat{i}\,F_x + \hat{j}\,F_y$$

$$\vec{F} = \hat{i}\,10\sin 60° + \hat{j}\,10\cos 60°$$

$F_x = 10 \times\sqrt{3}/2 = 5\sqrt{3}$ N and $F_y = 10\times1/2 = 5$ N

We have no net force in the vertical direction.

So, $R = 10\cos 60° + 4g = 5 + 40 = 45$ N

The net unbalanced force in the horizontal direction is

$$F_x = ma, \qquad \text{So, } a = \frac{5}{4}\sqrt{3}\text{ m/s}^2$$

A.1.2.3 (C)

Suppose X is a 5 kg block and Y is a 10 kg block placed as shown in Fig. A.1.2.2(a)

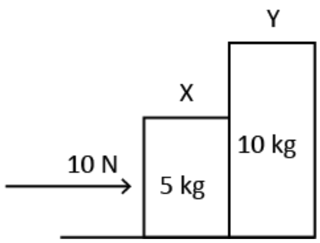


Fig. A.1.2.2(a)

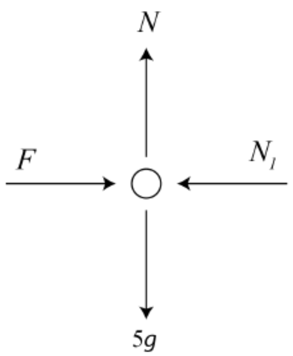


Fig. A.1.2.2(b)

FBD for X

(Refer Fig. A.1.2.2(b))

N_1 is normal reaction of Y on X

$$N = 5g \quad ,$$

$$F - N_1 = 5a \tag{1.2.3.1}$$

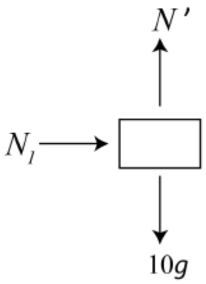


Fig. A.1.2.2(c)

FBD for Y

(Refer Fig. A.1.2.2(c))

$$N_1 = 10a \tag{1.2.3.2}$$

Adding equations (1.2.3.1) and (1.2.3.2),

$$F = (5 + 10)\, a$$

$$10 = 15a$$

$a = 10/15$ m/s^2 = $\frac{2}{3}$ m/s^2

Normal reaction of X on Y or Y on X

$N_1 = 10 \times 10/15 = 20/3$ N

A.1.2.4 (D) OX is the original direction of the ball. With symbols having their usual meanings,

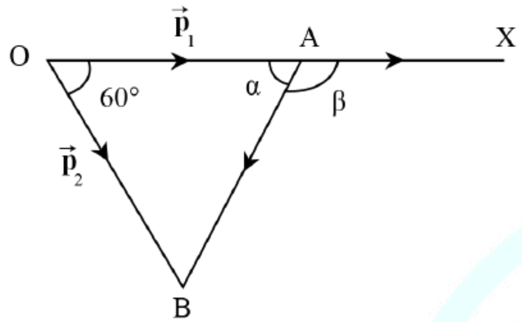


Fig. **A.1.2.3**

$\vec{OA} = \vec{p}_1$, $\vec{OB} = \vec{p}_2$

$\triangle\vec{p} = \vec{OB} - \vec{OA} = \vec{AB}$

AB^2 + OB^2− 2 OA.OB cos ∠AOB

$= p_1^2 + p_2^2 - 2p_1p_2$ cos 60°

$p_1 = p_2 = p =$ (160 g) (120 km/h)

= (0.16 kg) (120× $\frac{5}{18}$ m/s)

= $\frac{16}{3}$ kg m/s

AB^2 = $2p^2$(1- cos 60°) = $2p^2(1 - \frac{1}{2}) = p^2$

∴ AB = p

Let ∠ AOB = α

$\therefore \frac{OB}{\sin \alpha} = \frac{AB}{\sin 60°}$

But, OB = AB = p

∴ sin α = sin 60°, α = 60°

∠ OAX = β = 180°− 60° = 120°

A.1.2.5 (B)

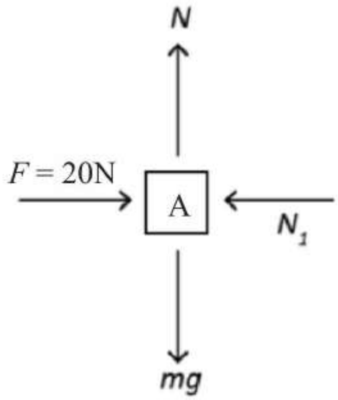


Fig. A.1.2.4 (a)

FBD for A [Fig. A.1.2.4 (a)].

a = Acceleration along the horizontal direction

$$20 - N_1 = 5a \quad (1.2.5.1)$$

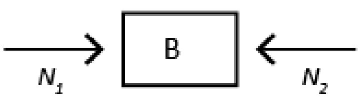


Fig.A.1.2.4(b)

FBD for B [Fig. A.1.2.4 (b)].

$$N_1 - N_2 = 10\,a \quad (1.2.5.2)$$

(Since three blocks behave as one)

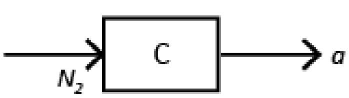


Fig.A.1.2.4(c)

FBD for C [Fig. A.1.2.4 (c)].

$$N_2 = 15a \quad (1.2.5.3)$$

Adding (1.2.5.1), (1.2.5.2) and (1.2.5.3)

$20 = 5a + 10a + 15a$

$a = 20/30\ \mathrm{m/s^2} = 2/3\ \mathrm{m/s^2}$

$$N_2 = 15 \times 2/3 = 10\ \mathrm{N}$$

$$N_1 = 10 \times \frac{2}{3} + 10 = \frac{50}{3}\ \mathrm{N}$$

A.1.2.6 (D)

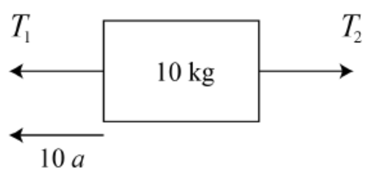


Fig.A.1.2.5(a)

FBD for Block A [Fig.A.1.2.5(a)]

$$T_1 - T_2 = 10a \tag{1.2.6.1}$$

FBD for Block B [Fig.A.1.2.5(b)]

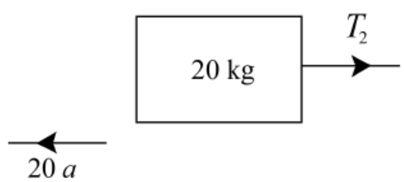


Fig.A.1.2.5(b)

$$T_2 = 20a \tag{1.2.6.2}$$

Adding Eq.(1.2.6.1) and (1.2.6.2),

$T_1 = 30a$ (since $T_1 = 10$ N)

10 N = 30a

$$a = \frac{1}{3} \text{ m/s}^2$$

$$T_2 = 20 \times \frac{1}{3} = \frac{20}{3} \text{ N}$$

A.1.2.7 (A) Refer Fig.A.1.2.6.

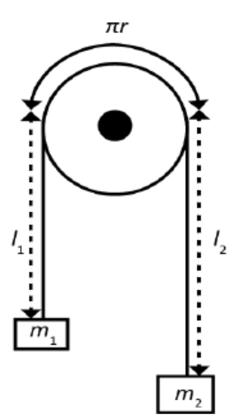


Fig.A.1.2.6

$$L = l_1 + l_2 + \pi r$$

$$\frac{dL}{dt} = \frac{dl_1}{dt} + \frac{dl_2}{dt} + \frac{d(\pi r)}{dt}$$

πr and L are constant. Therefore,

$$0 = v_1 + v_2$$

Or, $v_1 = \ - v_2$

$\frac{dv_1}{dt} = \ - \frac{dv_2}{dt}$,

Or, $a_1 = \ - a_2$

A.1.2.8 (A) with symbol their usual having meaning ,

we have

$v^2 - \frac{2Rd}{m} = 0$

$\therefore\ v^2 = \frac{2Rd}{m}$, $v = \sqrt{\frac{2Rd}{m}}$

A.1.2.9 (C) Refer to Fig.A.1.2.7. Let R be the normal reaction. If the acceleration of the system is f upward, then the equation of motion of m is

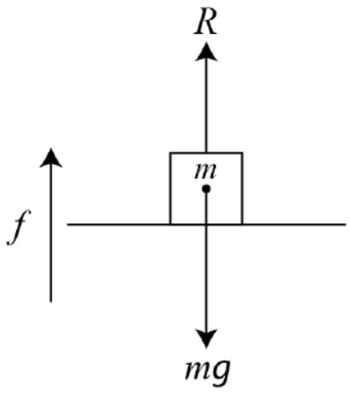


Fig.A.1.2.7

$R - mg = mf$

$R = m(g + f)$

When $f = 1.25\ \text{m/s}^2$, $R = (12.5)g$

$\therefore$ The required value of the mass is given by

$m = \frac{12.5 \times 10}{10 + 1.25} = \frac{12.5}{6.25} = 11\frac{1}{9} = 11.1$

A.1.2.10 (A) With symbols having their usual meanings, $v^2 = g^2 t^2$. So, the retarding force required to bring it to rest within a length $l = \frac{mv^2}{2l} = \frac{mg^2t^2}{2l}$. This retarding force will have to overcome the weight of the body. So, the required upward force $= mg + \frac{mg^2t^2}{2l} = mg\left(1 + \frac{g\ t^2}{2l}\right)$

A.1.3.1 (A) Refer [Fig.A.1.3.1]

The component of the force that tends to bring it down the incline

$F_d = mg\sin 53^o = 5\times10\times\frac{4}{5} = 40$ N

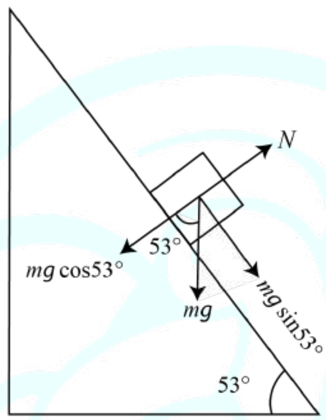


Fig.A.1.3.1

A.1.3.2 (B) Refer [Fig.A.1.3.1].

Force of limiting friction

$F_l = \mu_s mg\cos 53^o = 0.2\times10\times10\times\frac{3}{5} = 12$ N

A.1.3.3 (C) Refer [Fig.A.1.3.1].

Force of limiting friction

$f_l = \mu_s mg\cos 53^o = 0.2\times2.5\times10\times\frac{3}{5} = 3$ N

The component of the force that tends to bring it down the incline

$f_d = mg\sin 53^o = 2.5\times10\times\frac{4}{5} = 20$ N

As $f_d > f_l$ the body will slide down along the incline with acceleration and friction will no longer be static, but kinetic. And the following equation will hold good-

$$f_k = \mu_k \times mg\cos 53^o = 0.1 \times 2.5 \times 10 \times \frac{3}{5} = 1.5\ \text{N}$$

To get the acceleration let's consider the following FBD

$$mg\sin 53^o - f_k = ma$$

$$a = \frac{20-1.5}{2.5} = 7.4\ \text{m/s}^2$$

A.1.3.4 (A) Refer [Fig.A.1.3.2]. The body is at rest on the plane and is on brink of sliding down.

F is the force required to keep the body at rest on an inclined plane.

$$F + f_l = mg\sin\theta = 5 \times 10 \times \frac{4}{5} = 40$$

$$f_l = \mu_s mg\cos 53^o = 0.2 \times 5 \times 10 \times \left(\frac{3}{5}\right) = 6\ \text{N}$$

F is the minimum force required to keep the body at rest. Obviously,

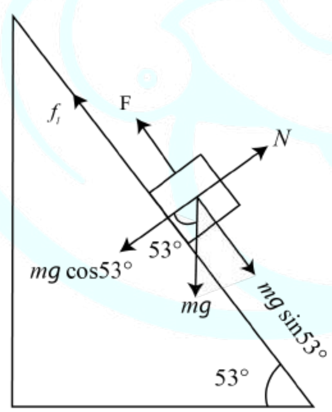


Fig. A.1.3.2

$$F = 40 - 6 = 34\ \text{N}.$$

A.1.3.5 (D) Refer [Fig.A.1.3.2].

Force applied to keep the object at rest $F = 62$ N

The component of the force that tends to bring the mass down the inclined plane

$$f_d = mg\sin 53^o = 10 \times 10 \times \frac{4}{5} = 80 \text{ N}$$

The force of friction

$$f_l = 80 - 62 = 18 \text{ N}$$

We have, $f_l = 18 = \mu_s mg \cos 53^o = \mu_s \times 5 \times 10 \times \frac{3}{5}$

$$\mu_s = \frac{18 \times 5}{10 \times 10 \times 3} = 0.3$$

A.1.3.6 (C). Refer [Fig.A.1.3.3].

Let F be the force required to move the body up with constant velocity. The net force on the body is zero. Therefore,

$F = f_k + mg \sin 53^o$

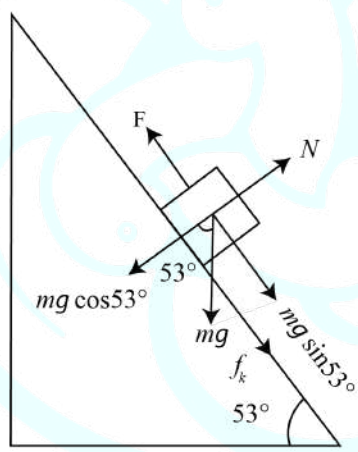


Fig.A.1.3.3

$f_k = \mu_k mg \cos 53^o = 0.2 \times 2.5 \times 10 \times \frac{3}{5} = 3 \text{ N}$

$mg \sin 53^o = 2.5 \times 10 \times \frac{4}{5} = 20 \text{ N}$

$$F = (3 + 20) \text{ N} = 23 \text{ N}$$

A.1.3.7 (C) Refer [Fig.A.1.3.3].

Let F be the force required in moving the body up the inclined with an acceleration of 1 m/s^2

The equation of motion is

$$F - (f_k + mg\sin 53^o) = ma$$

We have,

$$f_k = \mu_k \times mg\cos 53^o = 0.1 \times 5 \times 10 \times \frac{3}{5} = 3\text{ N}$$

$$\text{And } mg\sin 53^o = 5 \times 10 \times \frac{4}{5} = 40\text{ N}$$

$$\therefore\ F = 3 + 40 + (5 \times 1) = 48\text{ N}$$

A.1.3.8 (A).Refer [Fig.A.1.3.4].

Let F be the force required to move the body down the plane with a constant velocity.

We have,

$$F = mg\sin 53^o - f_k$$

Also,

$$f_k = \mu_k\, mg\cos 53^o = 0.1 \times 10 \times 10 \times \frac{3}{5} = 6\text{ N}$$

and

$$mg\sin 53^o = 10 \times 10 \times \frac{4}{5} = 80\text{ N}$$

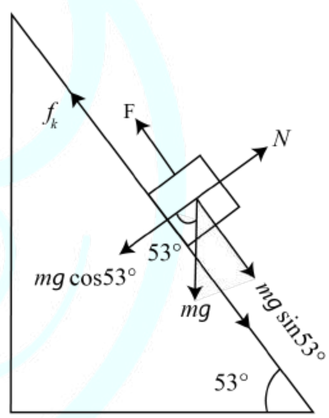


Fig.A.1.3.4

Therefore ,

$$F = (80 - 6)\text{ N} = 74\text{ N}$$

A.1.3.9 (B)

1. True, because for the two bodies to start moving together the minimum value of $F = 0.15\ mg$.
2. True, because the limiting value of F beyond which A will slip over B is $0.45\ mg$.
3. False, because for values of $F > 0.45\ mg$ N, A will slip over B True , because the minimum value of F for which the two blocks start moving is $F = 0.15\ mg$.

4. True, because $0.45\ mg$ happens to be the maximum value of F for which there is no relative motion between A and B.

The solution is given below with the help of Fig.A.1.3.5.

$F_{sB} =$ The limiting frictional force acting on B when A does not move with respect to B.

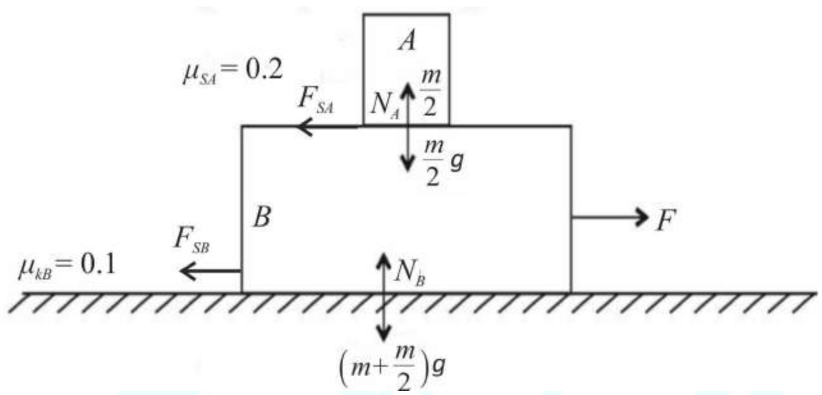


Fig.A.1.3.5

$$F_{sB} = \mu_{sB}N_B = 0.1\left(m + \frac{m}{2}\right)g$$

$$= \frac{0.3}{2}mg = 0.15\ mg$$

$$F_{sA} = \text{Limiting force of friction on A} = \mu_{sA}N_A$$

$$= 0.2\left(\frac{m}{2}\right)g = 0.1\ mg$$

The minimum value of F for system to move = 0.15 mg

For A to remain at rest w.r.t. B, A should move with the same acceleration as B .For A not to slip the maximum acceleration a_A will be due to the limiting static frictional force acting on A.

Therefore,

$$\frac{m}{2}a_A = F_{SA} = 0.1\ mg$$

$$\therefore a_A = 0.2\ g\ \text{m/s}^2$$

If A moves with an acceleration greater than a_A then A will slip over block B.

From FBD of block B,the net force acting on B is

$f = F - F_{sA} - F_{sB}$

$f = F - 0.15\, mg - 0.1\, mg = ma_B$

A is stationary w.r.t. B. if $a_A = a_{B.}$

Therefore,

$m \times 0.2g = F - 0.25\, mg$

Or, $F = 0.45\, mg$

This is the maximum value of F so that there is no relative motion between A and B.

A.1.3.10 (A)

(A) Correct. The net force is zero, which is the condition for motion with uniform speed. $\tau = 0$, which means that the net torque is zero, ensures that there would not be any relative motion between different parts of the body about P.

(B) False, if there is a non-zero torque at point P then there will be motion between different parts of the body.

(C) False, if forces are not coplanar then there will be a net force acting, and the body will not move with uniform speed.

(D) False, both one and two are essential conditions for a body to move with uniform speed.

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2. Work, Energy and Power

Module 2.1 Work and Power

Q.2.1.1 A cyclist, while in motion, applies brakes and comes to rest over a distance of 10 m. Assuming that the average force exerted by the road on the cycle to be 500 N, The work done (i) by the road on the cycle and (ii) by the cycle on the road are respectively equal to

(A) − 5000 J and + 5000 J

(B) + 5000 J and − 5000 J

(C) − 5000 J and zero

(D) zero and − 5000

Q.2.1.2 A body of mass 2 kg is initially at rest, and it moves under the action of an applied horizontal force of 18 N on a horizontal table. The coefficient of kinetic friction between the body and the table is 0.1. Taking g = 10 m/s^2, the work done by the applied force and the force of friction on the body, in 10 s are respectively equal to

(A) 7200 J and − 800 J

(B) − 7200 J and + 800J

(C) 8100 J and − 900J

(D) − 8100 J and + 900J

Q.2.1.3 A variable force $\vec{F}(t)$, acts on an object of mass *m,* initially at rest. The instantaneous power, associated with the force, varies as $t^{\frac{3}{2}}$.

$\left|\vec{F}(t)\right|$ i.e., the dependence of $\left|\vec{F}\right|$, on time, can be expressed as

(A) $\left|\vec{F}(t)\right|$ = (a constant) $(t^{\frac{1}{2}})$

(B) $\left|\vec{F}(t)\right|$ = (a constant) $(t^{\frac{1}{4}})$

(C) $\left|\vec{F}(t)\right| =$ (a constant) $(t^{\frac{3}{2}})$

(D) $\left|\vec{F}(t)\right| =$ (a constant)$(-\ t^{\frac{3}{4}})$

Q.2.1.4 A particle is moved from point P (1, 2, 3) to point Q (3, 4, 5) by the application of a constant force $\vec{F}$ given by $\vec{F} = \left(-\ 2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ newton. The work done by the force on the particle is

(A) 4 J

(B) 6 J

(C) 8 J

(D) 10 J

Q.2.1.5 An object of mass *m* is raised from the surface of the earth to a height *H*. If the height *H* is comparable to the radius *R* of the earth, the work done, against the force of gravity, is

(A) $m\frac{GM}{R^2}H$

(B) $m\frac{GMH}{R^2}\left(1 + \frac{H}{R}\right)^{-2}$

(C) $m\frac{GMH}{R^2}\left(1 + \frac{H}{R}\right)^{-1}$

(D) $m\frac{GMH}{R^2}\left(1 + \frac{H}{R}\right)$

Q.2.1.6 An object of mass 100 kg is raised from the surface of the earth (assume to be a sphere of radius ≃ 6400 km) to a point at a height of 3200 km above the surface. Taking $g = 10 \text{m/s}^2$, the work done by the gravitational force in raising this object is

(A) 3.2×10^9 J

(B) $-\ 3.2 \times 10^9$ J

(C) $-\ 2.13 \times 10^9$ J

(D) 2.13×10^9 J

Q.2.1.7 A body is moving in a particular direction under the influence of a source of constant power supplying energy. If it starts from rest from its origin, then which of the following represents the correct variation of displacement as a function of time for its motion?

(A) $s \propto t$

(B) $s \propto t^{3/2}$

(C) s = a constant

(D) $s \propto t^{2/3}$

Q.2.1.8 A body of mass 0.5 kg travels in a straight line with velocity $v = a\, x^{3/2}$ where $a = 5\ \text{m}^{-1/2}\ \text{s}^{-1}$ and x is the displacement of the body. The work done by the net force, during its displacement from $x = 0$ to $x = 2$ m, is

(A) 1.5 J

(B) 50 J

(C) 10 J

(D) 100 J

Q.2.1.9 An object of mass m initially at rest, falls from a height H ($H \ll R$, R = radius of the earth), under the influence of the gravitational force of the earth and the opposing resistive force due to earth's atmosphere. Assuming this resistive force to be directly proportional to the instantaneous speed (v) of the particle ($F_{resistive} = kv$), the work done by the net force, on the object, when it reaches the surface of the earth, is [Hint: As a first approximation we may take velocity $v \approx \sqrt{2gx}$]

(A) $mgH - \frac{2}{3}\sqrt{2gk^2H^3}$

(B) $mgH - \frac{2}{3}\sqrt{gk^2H^3}$

(C) $mgH - \frac{3}{2}\sqrt{gk^2H^3}$

(D) $mgH + \frac{2}{3}\sqrt{2gk^2H^3}$

Q.2.1.10 An object of mass m, initially at rest at the origin, is acted upon by a variable force $\vec{F}(t)$, where $\vec{F}(t) = kt\hat{i}$ The instantaneous displacement $\vec{x}(t)$, of the object, and the instantaneous power $P(t)$, associated with the force $\vec{F}(t)$, would then be given, respectively, by

(A) $\left(\frac{k}{2m}t^2\right)\hat{i}$ and $\frac{k^2}{2m}t^3$

(B) $\left(\frac{k\,t^2}{2m}\right)\hat{i}$ and $\frac{k^2}{4m}t^3$

(C) $\left(\frac{k}{m}\ \frac{t^3}{6}\right)\hat{i}$ and $\frac{k^2}{2m}t^3$

(D) $\left(\frac{k}{m}\ \frac{t^3}{6}\right)\hat{i}$ and $\frac{k^2}{4m}t^3$

Module 2.2 Work Energy Theorem

Q.2.2.1 A ball of mass m when pushed down from the point P, with a speed v, just manages to reach the top end Q of the smooth hemispherical bowl of radius R (Fig.Q.2.2.1)

. The velocity v is equal to

(A) $\sqrt{2g(R)}$

(B) $\sqrt{2g(R - h)}$

(C) $\sqrt{2g(R + h)}$

(D) $\sqrt{2gh}$

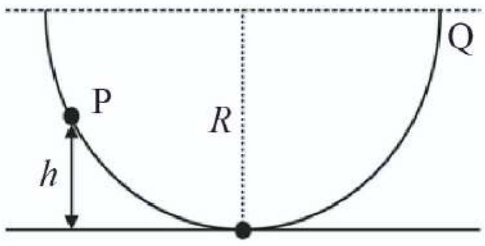


Fig.Q.2.2.1

Q.2.2.2 An arrow of mass m is shot from a bow, whose string exerts an average force F on the arrow over a length l . The speed of arrow as it leaves the bow is

(A) $(\frac{Fl}{m})^{\frac{1}{2}}$

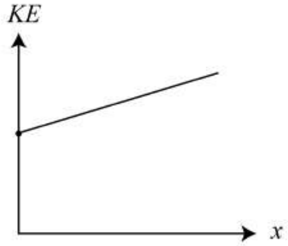
(B) $(\frac{2Fl}{m})^{\frac{1}{2}}$

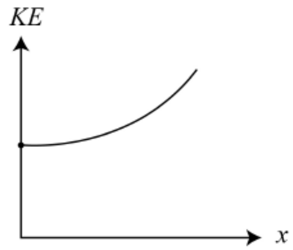
(C) $(\frac{Fl}{2m})^{\frac{1}{2}}$

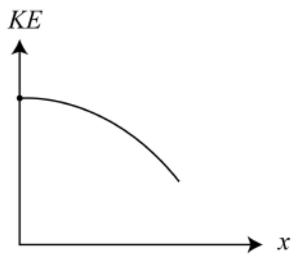
(D) $2\,(\frac{Fl}{m})^{\frac{1}{2}}$

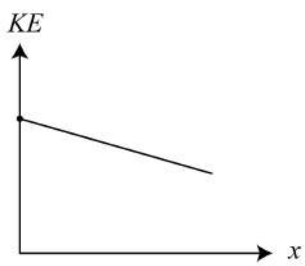
Q.2.2.3 A particle is sliding down an inclined plane having a uniform dynamic friction, from a height h. The variation in Kinetic Energy with x which is the distance through which the particle moves is given by which one among the following?

(A)



(B) 

(C) 

(D) 

Q.2.2.4 A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table. The coefficient of kinetic friction between the body and the table is 0.1.

Using $g = 10\ ms^{-2}$, the

(i) work done by the applied force in 10 s,

(ii) work done by the force of friction in 10 s,

(iii) work done by the net force on the body in 10 s,

(iv) change in kinetic energy of the body in 10 s,

are respectively equal to

(A) 875 J, − 250 J, 625 J, 625 J

(B) − 875 J, +250 J,− 625 J, 625 J

(C) 1225 J, − 350 J, 875 J, 875 J

(D) − 1225 J, +350 J, − 875 J, 875 J

Q.2.2.5 When a cloud mass at a height of 5 km above the ground condenses, it inundates an area of 10^5 m^2 with ankle deep (10 cm) water. If $g = 10$ ms^{-2}, and density of water is 1 gcm^{-3}, the loss of potential energy (in joules) of the cloud mass is 5×10^n where n is equal to

(A) 8

(B) 9

(C) 10

(D) 11

Q.2.2.6 A rain drop of radius 2 mm falls from a height of 500 m above the ground. Due to the viscous resistance of the air, it falls with decreasing acceleration and soon attains a uniform terminal speed. It reaches the ground with a speed of 10 ms^{-1}.

Taking $g = 9.8$ ms^{-2}, the work done by the gravitational and the resistive forces on the drop, are respectively (nearly) equal to

(A) − 0.1642 J and +0.0017 J

(B) + 0.1642 J and −0.1625 J

(C) + 0.1642 J and −0.0017 J

(D) − 0.1642 J and +0.1625 J

Q.2.2.7 A particle starts with a non-zero initial velocity and moves along a straight line with a uniform acceleration. The plot of the variation of its kinetic energy (*KE*) with time will have an intercept on the KE-axis and will be

(A) a straight line inclined at an acute angle with the time axis

(B) a straight line inclined at an obtuse angle with the time axis

(C) a parabola with positive slope at its point of intersection with the KE axis

(D) a parabola with zero slope at the point of intersection with the KE axis

Q.2.2.8 A simple pendulum, having a length of 1.5 m, has its bob pulled to a horizontal position (Fig.Q.2.2.2). 5% of the total energy of the bomb dissipates as it comes down to its lowest position.

Taking $g = 9.8$ ms^{-2}, speed of the bob when it (first) comes back at its lowest position would be (nearly)

(A) 5.42 ms^{-1}

(B) 5.28 ms^{-1}

(C) 5.15 ms^{-1}

(D) 5.08 ms^{-1}

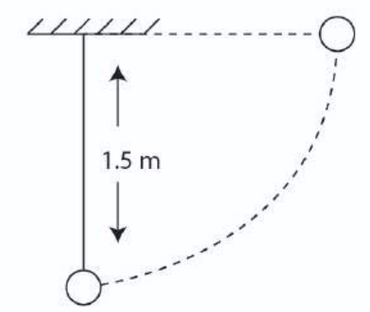


Fig.Q.2.2.2

Q.2.2.9 The electron, in a hydrogen atom, is orbiting around the nucleus in a circular orbit of radius r with a speed v. The needed centripetal force $\left(\frac{mv^2}{r}\right)$ is provided by the attractive coulomb force $(= k.\frac{e.e}{r^2})$ between the electron and the proton of the hydrogen nucleus.

If the radius of the electron orbit were to decrease to $r/2$, the kinetic energy of the electron would

(A) increase by an amount of (ke^2/r)

(B) increase by an amount of $(ke^2/2r)$

(C) decrease by an amount of (ke^2/r)

(D) decrease by an amount of $(ke^2/2r)$

Q.2.2.10 It is well known that a raindrop (assumed to be at rest, to start with) falls under the influence of the downward gravitational force and the opposing resistive force. The latter is known to be proportional to the speed of the drop but is otherwise undetermined. Consider a drop of mass 1.00 g falling from a height of 1.00 km. It hits the ground with a speed of 50.0 ms^{-1}. Taking $g = 10$m/s^2 work done by the gravitational force and the work done by the unknown resistive force are respectively equal to

(A) – 10.0 J and + 8.75 J

(B) + 10.0 J and – 8.75 J

(C) + 10.0 J and – 1.25 J

(D) – 10.0 J and + 1.25 J

Module 2.3 Potential Energy and Energy of Spring

Q.2.3.1 A construction worker has to supply bricks to his partner at a height of 3 m vertically above him. He throws them up so that they reach his partner with a velocity of 2 ms^{-1}. What percentage of his work could be saved if he threw them away so that they might just reach his partner? (Take $g = 10 \ ms^{-2}$)

(A) 5
(B) 6.25
(C) 7.5
(D) 8

Q.2.3.2 Two springs, (1) and (2), have spring constant k_1 and k_2, respectively, with $k_1 < k_2$ The springs are stretched
(a) by applying equal force on them and
(b) so that their lengths increase by the same amount.
Let the work done on the two springs, in cases (a) and (b), be (W_{1a} and W_{2a}) and (W_{1b} and W_{2b}), respectively. We would then have

(A) $(W_{1a} > W_{2a})$ and $(W_{1b} = W_{2b})$
(B) $(W_{1a} = W_{2a})$ and $(W_{1b} < W_{2b})$
(C) $(W_{1a} < W_{2a})$ and $(W_{1b} > W_{2b})$
(D) $(W_{1a} > W_{2a})$ and $(W_{1b} < W_{2b})$

Q.2.3.3 Imagine a mass m to be moved from the centre of the earth to a point on the surface of the earth. The work done (by some external agency) against the gravitational pull of the earth, would then be equal to
(g = acceleration due to gravity on the surface of the earth)

(A) mgR
(B) $mgR/2$
(C) $mgR/3$
(D) $mgR/4$

Q.2.3.4 Let a conservative (inverse square) force do positive work on a particle while displacing it towards the source of this force. If the instantaneous distance, between the origin of the source of this force and the particle is r, we say that the potential energy of the particle

(A) decreases with a decrease in r, and it is taken as zero as $r \to 0$

(B) decreases with a decrease in r, and it is taken as zero as $r \to \infty$

(C) increases with a decrease in r, and it is taken as zero as $r \to 0$

(D) increases with a decrease in r, and it is taken as zero as $r \to \infty$

Q.2.3.5 A given spring, when compressed by an amount x_0, can cause an object of mass m, kept on it, to get 'thrown up' through a maximum vertical height H. The spring constant, k, of this spring, equals

(A) $\frac{mgh}{x_0^2}$

(B) $\frac{mgh}{2x_0^2}$

(C) $\frac{2mgh}{3x_0^2}$

(D) $\frac{2mgh}{x_0^2}$

Q.2.3.6 Four springs (1, 2, 3, and 4) are made from the same material but

(i) Springs (1) and (2) have the same diameter but different lengths, say l_1 and l_2, with $l_1>l_2$.

(ii) Springs (3) and (4) have the same length but different diameters, say d_1 and d_2, with $d_1>d_2$.

The potential energies, stored in the four springs, when they are stretched by equal amounts (say x each) are U_1, U_2, U_3 and U_4 respectively. We would then have

(A) $U_1 < U_2$ and $U_3 < U_4$

(B) $U_1 > U_2$ and $U_3 > U_4$

(C) $U_1 < U_2$ and $U_3 > U_4$

(D) $U_1 > U_2$ and $U_3 < U_4$

Q.2.3.7 A particle is moving along the x-axis under the action of a variable force $F(x)$. The PE associated with the particle, is then expressed by the function $U(x)$. if $F(x)$ is conservative then which among the following is/are possible?

Ⅰ. $F(x) = -\frac{du}{dx}$

Ⅱ. $F(x) = -\frac{du}{dx}$

(A) Only Ⅰ

(B) Only Ⅱ

(C) Both Ⅰ and Ⅱ

(D) Neither Ⅰ , nor Ⅱ

Q.2.3.8 A given charged particle, having a charge q, is moved from the origin, to the point P (4, 5), via three different paths (1), (2), (3), as shown, in the *XY* plane (Fig.Q.2.3.2). Let there be a uniform electric field $\vec{E}$, in the *XY* plane given by $\vec{E} = C_1\hat{i} + C_2\hat{j}$, where (C_1, C_2) are constants having the same dimensions as that of the electric field.

Let the work done, on the charged particles, by the electric field in the three cases, be W_1, W_2 and W_3 respectively. A direct calculation of W_1, W_2, W_3 would then show that

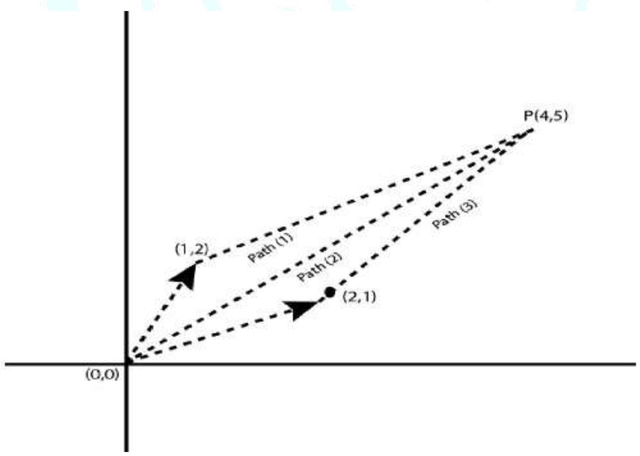


Fig.Q.2.3.2

(A) $W_1 \neq W_2 = W_3$; the force is non-conservative

(B) $W_2 \neq W_3 = W_1$; force is non-conservative

(C) $W_3 \neq W_1 = W_2$; force is non-conservative

(D) $W_1 = W_2 = W_3$; force is conservative

Q.2.3.9 It is known that comets move around the Sun in elliptical orbits of very high eccentricity. In general, the instantaneous gravitational force, on the comet, due to the Sun is not normal to its instantaneous velocity. The net work done, by the gravitational force of the Sun, on the comet, over one complete orbit, of the comet, would then be

(A) A non-zero positive work; dependent on the eccentricity of the orbit.

(B) A non-zero negative work; dependent on the eccentricity of the orbit.

(C) Zero only if the elliptical orbit approaches a circular shape.

(D) Zero; irrespective of the eccentricity of the orbit.

Q.2.3.10 Ernst Rutherford, in his nuclear model of the hydrogen atom, assumed that the electron orbits around the nucleus in a circular orbit of radius r, the required centripetal force being provided by the electric force of attraction between the electron and the nucleus.

This motion of the electron around the nucleus is therefore an accelerated motion and as per the electromagnetic theory the electron must continuously radiate energy as it orbits around the nucleus. This continuous loss of energy would then cause the electron to continuously move

(A) away from the nucleus with a continuous decrease in its kinetic energy.

(B) towards the nucleus with a continuous decrease in its kinetic energy

(C) towards the nucleus with a continuous increase in its kinetic energy.

(D) away from the nucleus with a continuous increase in its kinetic energy.

Module 2.4 Elastic and Inelastic Collision

Q.2.4.1 A gas molecule of mass m moving with velocity v undergoes an elastic collision with the wall of the container as shown schematically in Fig.Q.2.4.1. If the time of impact of collision is τ. Then the force exerted by the molecule on the wall is given by

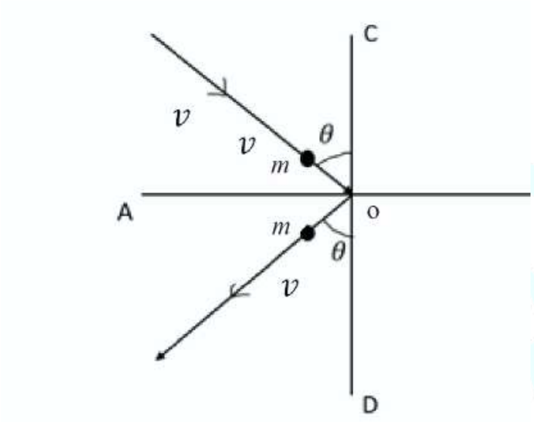


Fig.Q.2.4.1

(A) $2mv \sin\theta/\tau$ along OA

(B) $2mv \cos\theta/\tau$ along OA

(C) $2mv \sin\theta/\tau$ along AO

(D) $2mv \cos\theta/\tau$ along AO

Q.2.4.2 Three objects of masses m, m and $2m$ are kept on a smooth horizontal floor (Fig. Q.2.4.2). The spring constant of the spring, connected between objects (2) and (3) is k.

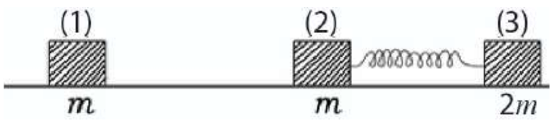


Fig.Q.2.4.2

The object (1), is set moving with a velocity v_0 and undergoes an elastic collision with object (2) at time $t = 0$.

At some instant ($t = t_0$, say), after the collision, the spring is compressed by an amount x_0 and the instantaneous velocities of objects (2) and (3), are equal to each other.

The relation, between k, v_0 and x_0, is

(A) $k = \frac{mv_0^2}{x_0^2}$

(B) $k = \frac{1}{2}\frac{mv_0^2}{x_0^2}$

(C) $k = \frac{2}{3}\frac{mv_0^2}{x_0^2}$

(D) $k = \frac{3}{4}\frac{mv_0^2}{x_0^2}$

Q.2.4.3 A bob of mass M is suspended from a wall, on a small peg, using an ideal string of length L. The bob is pulled up to a horizontal position and 'let-go'. It hits the wall, rebounds, rises up and again comes back to hit the wall (Fig.Q.2.4.3). The coefficient of restitution for the bob-wall collision equals $\frac{2}{\sqrt{5}}$.

It is given that $\log\sqrt{5} = 0.35$ and $\log 2 = 0.3$. Then after nearly how many collisions will the angular amplitude (θ) of the bob be 60^0?

(A) 2

(B) 3

(C) 4

(D) 5

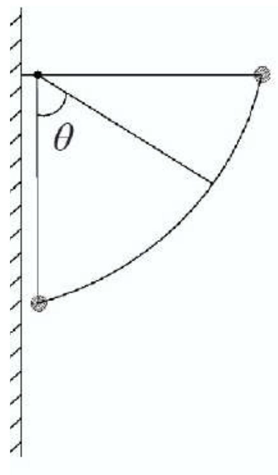


Fig.Q.2.4.3

Q.2.4.4 A ball initially at rest, is dropped (at $t = 0$) from a height H onto the floor. If the coefficient of restitution between the ball and the floor is e, the time instant, at which the ball would be just making its second impact on the floor, would be

(A) $\sqrt{\frac{2H}{g}}(1 + 2e)$

(B) $\sqrt{\frac{2H}{g}}(1 - 2e)$

(C) $\sqrt{\frac{2H}{g}}(1 - e)$

(D) $\sqrt{\frac{2H}{g}}(1 + e)$

Q.2.4.5 Two balls, of equal masses (m each), are kept on a smooth table (Fig.Q.2.4.4). One of these balls is given a velocity $\vec{v}$ and made an elastic collision with the other ball which is at rest. The two balls move along directions inclined at angles θ_1 and θ_2 to the initial direction of $\vec{v}$ is as shown in the figure. If $\theta_1 = \frac{\pi}{6}$, then θ_2 is equal to

(A) $\frac{\pi}{12}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{5\pi}{12}$

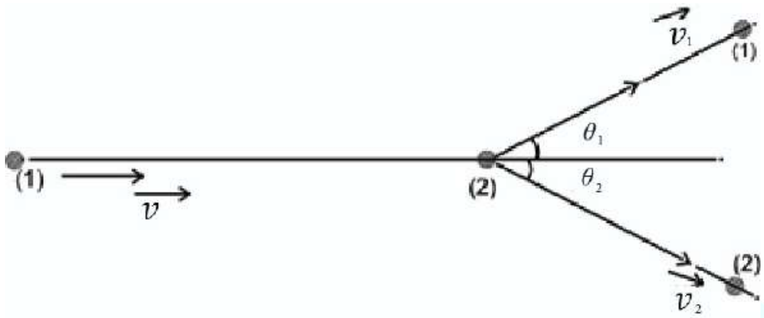


Fig.Q.2.4.4

Q.2.4.6 A ball falls from a height of 200 cm onto a rough floor and rises back after impact to a height of 162 cm. The coefficient of restitution between the ball and the floor, and the height to which the ball rises after the second impact, are respectively (nearly) equal to

(A) 0.9 and 131 cm

(B) 0.81 and 131 cm

(C) 0.81 and 106 cm

(D) 0.9 and 106 cm

Q.2.4.7 A ball initially at rest is dropped vertically from a height of H onto a floor. If the coefficient of restitution between the ball and the floor equals e. The height to which the ball would rise after its third impact, would be

(A) $e^3 H$

(B) $e^6 H$

(C) $e^2 H$

(D) $e^4 H$

Q.2.4.8 In the game of cricket, a player hits a ball of mass 0.16 kg horizontally square of the wicket, which means perpendicular to the direction of the ball which was also traveling horizontally at the time of impact with the bat. The ball was traveling at a speed of 144 km/h when it was hit and the speed acquired by the ball was 270 km/h immediately after the impact. If the change in momentum (in kg m/s) and its direction are Δp and θ respectively, then their values are:

(A) 13.6 kg m/s, at an angle of $(\frac{\pi}{2} + \tan^{-1}\frac{8}{15})$ with the original direction of the ball

(B) 13.6 kg m/s, at an angle of $(\tan^{-1}\frac{8}{15})$ with original direction of ball

(C) 5.6 kg m/s, at an angle of $(\tan^{-1}\frac{8}{15})$ with original direction of ball

(D) 5.6 kg m/s, at an angle of $(\frac{\pi}{2} + \tan^{-1}\frac{8}{15})$ with original direction of ball

Q.2.4.9 Two particles P and Q, each of mass m start simultaneously from point A on a frictionless circular track and move in opposite directions with velocities having their magnitudes in the ratio of 3:2 (Fig.Q.2.4.5). If the two particles collide at a point such that particle P comes to rest instantaneously, then what will be the component of change in momentum of particle Q along OA.

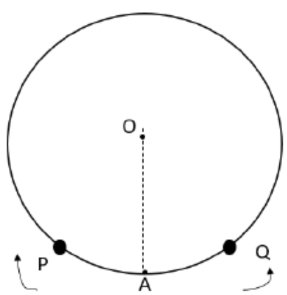


Fig.Q.2.4.5

(A) $5\,mu\,\cos\,54°$

(B) $mu\,\sin\,54°$

(C) $mu\,\sin\,36°$

(D) $\sqrt{13}\,mu\,\cos\,36°$

Q.2.4.10 A bullet of mass m moving with a velocity u strikes a greater mass M at rest and gets embedded into it. The composite mass starts moving with a velocity v. There will be a decrease in kinetic energy, and the difference can be expressed as being equivalent to the KE of a mass m_0 moving with a velocity w, where

(A) $m_0 < m$ & $w = v$

(B) $m_0 > M$ & $v < w < u$

(C) $m_0 < m$ & $w = u$

(D) $m < m_0 < M$ & $w = v$

ANSWERS

Module 2.1 Work and Power

A.2.1.1 (C) The force 500N acts in a direction opposite to the motion of the bicycle .So work done by the road = (−500N) (10m) = − 5000 J. The road does not move .So the work done on the road is zero.

Option (A), if selected, would be incorrect as here, the student is assuming the validity of Newton's third law for the two cases of work also. The work done by the cycle, on the road, is zero as the cycle does not cause any displacement of the road.

In Option (B), we have the additional error of attaching a +ve sign to the work in the first case. The work, done by the road on the cycle, is a negative work, as the force exerted by the road on the cycle is directed opposite to its displacement.

In Option (D), Work in the first case has been incorrectly taken as zero and in the second case work (which is zero) has been given a finite value.

A.2.1.2 (A)

Force of friction = $\mu N = \mu mg$

$= 0.1 \times 2 \times 10$ N = 2 N

$\therefore$ Net force on the body = 18 N − 2 N =16 N

Net acceleration = 16 N/2 kg= 8 m/s^2

$\therefore$ Distance moved in 10 s = $0 + \frac{1}{2}at^2 = 0 + \frac{1}{2}\times 8\times 10^2 = 400$ m

$\therefore$ Work done by the applied force= 18 N × 400 m = 7200 J

Work done by the force of friction = −2 N×400 m = − 800 J.

A.2.1.3 (B)

We have,

Instantaneous power $= \vec{F}.\vec{v}$

$$= \vec{F}.\left[\frac{\vec{F}}{m}t\right] = \frac{t}{m}F^2$$

$$\therefore \frac{t}{m}F^2 = kt^{\frac{3}{2}}$$

Where k = a constant

$$\therefore F^2 = (km)t^{\frac{1}{2}} = ct^{\frac{1}{2}}$$

Where $c = km$ = a constant

Hence , $\left|\vec{F}(t)\right| =$ (a constant) $t^{\frac{1}{2}}$

The other Options, if selected, would be due to an incorrect use of the formula for instantaneous power.

A.2.1.4 (D)

We have, for a constant force, $W = \vec{F}.\vec{S}$ Here $\vec{F} = \left(-2\hat{i} + 3\hat{j} + 4\hat{k}\right)$

Also $\vec{s} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} = (3 - 1)\hat{i} + (4 - 2)\hat{j} + (5 - 3)\hat{k} = 2\hat{i} + 2\hat{j} + 2\hat{k}$

$$\therefore W = \vec{F}.\vec{s} = (-4 + 6 + 8) = 10 \text{ units}$$

A.2.1.5 (C)

Here, $W = \int_0^H \frac{GMm}{(R+x)^2}dx = \left|-\frac{GMm}{(R+x)}\right|_0^H = GMm\left[\frac{1}{R} - \frac{1}{R+H}\right] = \frac{GMmH}{R(R+H)} = m.\frac{GMH}{R^2}\left(1 + \frac{H}{R}\right)^{-1}$

A.2.1.6 (C) It is given that, R = 6400 km. So, 3200 km = $\frac{R}{2}$. So, the object is being raised from $x = R$ to $x = R + \frac{R}{2} = \frac{3R}{2}$

$$W =- \int_{x=R}^{x=\frac{3}{2}R} \frac{GMm}{x^2} dx = GMm\left| - \frac{1}{x}\right|_{\frac{3}{2}R}^{R} = GMm\left|\frac{2}{3R} - \frac{1}{R}\right| =- GMm\left(\frac{1}{3R}\right) =- \frac{GM}{R^2} m \frac{R^2}{3R} =- mg\frac{R}{3}$$

$W =- 100\times10\times\frac{6400}{3} \times 10^3$ J = $-$ 2.13 × 10^9 J

∴ Work by the gravitational force

$$\cong - 2.13 \times 10^9 \text{ J}$$

[Note: By using W = $m\,g\,H$, we get

W= 100 ×10×3200×10^3J = 3.2 × 10^9 J

This is incorrect as it does not take into account the variation of g with H.

Here H is not negligible compared to R]

A.2.1.7 (B)

$P = \vec{F}.\vec{v} =$ a constant $= k,$ say

$$\therefore \quad m\frac{d\vec{v}}{dt}.\vec{v} = k$$

$$\therefore \frac{1}{2}\frac{d}{dt}(v^2) = k$$

On $(v^2) = 2kt + A\,,\ A =$ a constant

It is given that , $v = 0$ at $t = 0$.

So, $A = 0$

Or $v^2 = 2kt$

$\therefore v = \sqrt{2k}\, t^{\frac{1}{2}}$

$\therefore \frac{ds}{dt} = \sqrt{2k}\, t^{\frac{1}{2}}$

$\therefore s = \sqrt{2k}\, t^{\frac{3}{2}} + B$, where $B =$ a constant

As $s = 0$ for $t = 0$, $B = 0$

$\therefore s = \sqrt{2k}\, t^{\frac{3}{2}}$

As per option (A), the displacement varies linearly with time, whereas displacement varies as $t^{\frac{3}{2}}$. Hence option (A) is incorrect.

For (B), the displacement varies with time as $t^{\frac{3}{2}}$. The graph should be a non-linear one. Also, its slope, given by (slope $= \frac{ds}{dt} = v$) should increase with time. Hence the option (B) is correct.

For (C), the displacement varies with time as $t^{\frac{3}{2}}$. The graph should be a non-linear one. But as per option (C) displacement remains constant with time. Hence option (C) is incorrect.

For (D), the displacement varies with time as $t^{\frac{2}{3}}$. The graph should be a non-linear one. Also, its slope is given by (slope $= \frac{ds}{dt} = v$) should increase with time. We therefore have to rule out option (D) as its slope is decreasing with time.

For option (B) the instantaneous slope is increasing with time as required.

A.2.1.8 (B)

$$\text{Here, } v = \left(\frac{dx}{dt}\right) = ax^{3/2}$$

$$\therefore \text{ Acceleration } = \frac{dv}{dt} = \frac{3}{2}ax^{1/2}\frac{dx}{dt}$$

$$= \frac{3}{2}ax^{1/2}(ax^{3/2})$$

$$= \frac{3}{2}a^2x^2$$

$$\therefore \text{ Force} = F(x) = \frac{3}{2}ma^2x^2$$

$$\text{Work done } = \int_{x=0}^{x=2} F(x)dx$$

$$= \frac{3}{2}ma^2\left|\frac{x^3}{3}\right|_0^2$$

$$= \frac{1}{2}ma^2\left|x^3\right|_0^2$$

$$= 4ma^2$$

$$= 4 \times 0.5 \times (5)^2 \text{ J} = 50 \text{ J}$$

A.2.1.9 (A)

$$F_{net} = mg - kv = mg - k(\sqrt{2gx})$$

$$dW = F(-dx) = [mg - k\sqrt{2gx}](-dx)$$

$$W = \int_H^0 F(-dx) = \int_H^0 [mg - k\sqrt{2gx}]dx = mgH - k\sqrt{2g}\left|\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^H = mgH - \frac{2}{3}\sqrt{2gk^2H^3}$$

A.2.1.10 (C)

The instantaneous acceleration $\vec{a}\,(t)$ of the object is given by

$$\vec{a}\,(t) = \frac{\vec{F}(t)}{m} = \left(\frac{k}{m}t\right)\hat{i}$$

$$\therefore \frac{d\vec{\upsilon}}{dt} = \left(\frac{k}{m}t\right)\hat{i}, \quad \therefore \vec{v}(t) = \left(\frac{k}{m}\frac{t^2}{2}\right)\hat{i}$$

$(\text{as } \vec{v}\,(0) = 0)$

Hence $\frac{d\vec{x}}{dt} = \left(\frac{k}{m}\frac{t^2}{2}\right)\hat{i}$

$$\therefore \vec{x}(t) = \left(\frac{k}{m}\frac{t^3}{6}\right)\hat{i} \qquad (\text{as } \vec{x}(0) = 0)$$

The instantaneous power, *P (t)*, is given by

$$P(t) = \vec{F}(t)\,.\,\vec{v}\,(t) = (k\;t\;\hat{i}\,)\;.\left(\frac{k}{m}\frac{t^2}{2}\right)\hat{i} = \frac{k^2}{2m}t^3$$

Module 2.2 Work Energy Theorem

A.2.2.1 (B)

Total energy of the ball, at the point $P = \frac{1}{2}mv^2 + mgh$. At the point Q, the ball has only PE, its KE, there, is zero as it (momentarily) comes to rest there.

$\therefore$ Total energy at $Q = mgR$

By the energy conservation principle,

$$\frac{1}{2}mv^2 + mgh = mgR$$

This gives, $v = \sqrt{2g(R - h)}$

A.2.2.2 (B)

Let the required speed $= u$.

We know from work energy theorem that the change in KE = Work done

Initial $KE = \frac{1}{2}mu^2$

Final $KE = 0$

$\therefore \frac{1}{2}mu^2 - 0 =$ (Average force) (length) $= Fl$

$\therefore \frac{1}{2}mu^2 = Fl$

Or $u^2 = \frac{2Fl}{m}$,

$$u = \sqrt{\frac{2Fl}{m}} = \left(\frac{2Fl}{m}\right)^{\frac{1}{2}}$$

A.2.2.3 (C) If the mass of the particle is m, and if the slope of the inclined plane is θ, and the uniform force of dynamic friction F is then the KE of the particle is given by, $KE = mgh\sin\theta - Fx$. So, the graph of KE vs. x is a straight line with a negative slope and the intercept with $x = 0$ is $mgh\sin\theta$ So, the key is Option (C)

A.2.2.4 (A)

Force of friction = 0.1×2×10 N = 2 N

Net Force = (7 − 2) N = 5 N

Net acceleration = $\frac{5}{2}$ ms^{-2} = 2.5 ms^{-2}

$$\therefore s_{10} = \left(\frac{1}{2}\times 2.5\times 10^{2}\right)\text{ m} = 125\text{ m}$$

∴ Works done are

a) (7× 125) J = 875 J

b) (− 2× 125) J = − 250 J

c) (875 – 250) J = 625 J

d) Change in kinetic energy = 625 J

A.2.2.5 (D) 10 cm = 0.1 m, 1 g cm^{-3} = $\left(\frac{10^{-3}}{10^{-6}} = 10^{3}\right)$ kg m^{-3}, 5 km = 5000 m

So, the required loss in PE = $10^{5} \times 0.1 \times 10^{3} \times 10 \times 5000$ J = 5×10^{11} J. So, $n = 11$

So, the correct option is (D).

A.2.2.6 (B)

Mass of the drop = $\frac{4\pi}{3}r^{3}\rho$

$$= \frac{4\pi}{3}\left(2\times 10^{-3}\right)^{3} \times 10^{3}\text{ kg}$$

$$= \frac{32\pi}{3} \times 10^{-6}\text{ kg}$$

∴ Work done by the gravitational force

$= mgh = \left(\frac{32\pi}{3} \times 10^{-6} \times 9.8 \times 500\right)$ J= $\left(\frac{160\pi}{3} \times 9.8 \times 10^{-4}\right)$ J=0.1642 J

The gain in KE of the drop

$= \left[\frac{1}{2}m(10)^2 - 0\right]$ J= $\frac{1}{2} \times \frac{32\pi}{3} \times 10^{-6} \times 10^2$ J $= \left(\frac{16\pi}{3}\right) \times 10^{-4}$ J $= 0.001675$ J

Hence, work done by the resistive force

$= -$ [(0.1642 $-$ 0.001675)] J

$= -$ 0.1625 J

A.2.2.7 (C) The kinetic energy is given by (with symbols having their usual meanings),

$KE = \frac{1}{2}m(u + ft)^2 = \frac{m}{2}(u^2 + 2uft + f^2t^2);$

$KE = E = a + bt + ct^2;\ E - a = c(t^2 + \frac{bt}{c}) = c\left\{\left(t + \frac{b}{2c}\right)^2 - \frac{b^2}{4c^2}\right\},$

Where $a = \frac{1}{2}mu^2, b = muf\ , c = \frac{1}{2}mf^2.$

$\therefore E - a + \frac{b^2}{4c} = c\left(t + \frac{b}{2c}\right)^2; E' = E - \left(a - \frac{b^2}{4c}\right) = c\left(t + \frac{b}{2c}\right)^2$

So, $E' = ct_1^2$, where $E' = E - E_0$, where $= E_0 = a - \frac{b^2}{4c}$ and $t_1 = t + \frac{b}{2c}$.

So, E' vs. t_1 curve will be a parabola with pole at $E = E_0$ and $t = -\frac{b}{2c}$ (Fig.A.2.2.1).

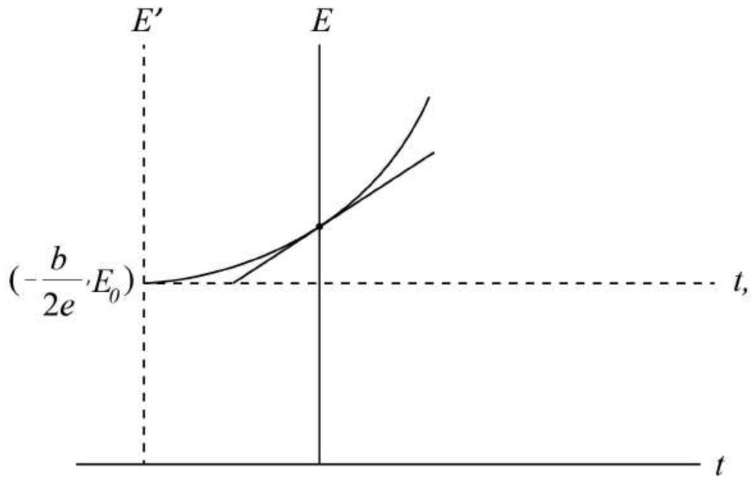


Fig.A.2.2.1

Since the parabola has a pole at $(-\frac{b}{2c}, E_0)$ The parabola will have zero slope at the pole, but it will intersect the E axis in the manner shown in Fig. 2.2.1, and at the point of intersection the tangent to the parabola will be making an acute angle with the positive direction with the time axis. Hence, the correct option is (C).

A.2.2.8 (B)

Initial PE of the bob $= mgh$

$= m \times 9.8 \times 1.5$ J = ($m \times 14.7$) J

Energy dissipated $= \frac{5}{100} \times (m \times 14.7)$ J = ($m \times 0.735$) J

∴ If v is the speed (when it (first) comes back) to its lowest position, we have

$$\frac{1}{2}mv^2 = m(14.7 - 0.735)$$

$$\therefore\ v^2 = 2 \times 13.965 = 27.930$$

$$\therefore\ v = \sqrt{27.930} = 5.28\ \text{ms}^{-1}$$

A.2.2.9 (B)

We have

$$\frac{mv_1^2}{r} = k.\frac{e^2}{r^2}$$

$$\therefore mv_1^2 = \frac{k\ e^2}{r} \therefore \text{initial } KE = \frac{1}{2}mv_1^2 = \frac{1}{2}\frac{k\ e^2}{r}$$

For the orbits of radius $r/2$, we would similarly have

$$\frac{mv_2^2}{(\frac{r}{2})} = k\frac{e^2}{(\frac{r}{2})^2}$$

$$\therefore mv_2^2 = \frac{k\ e^2}{\frac{r}{2}} = \frac{k\ e^2}{r}$$

$$\therefore \text{Final } KE = \frac{1}{2}mv_2^2 = \frac{k\ e^2}{r}$$

$$\therefore \text{ Change in } KE = \frac{k\ e^2}{r} - \frac{k\ e^2}{2r} = \frac{k\ e^2}{2r}$$

The KE, therefore, increases by $\left(\frac{ke^2}{2r}\right)$

A.2.2.10 (B)

The change in kinetic energy of the drop is

$$K = \left(\frac{1}{2}mv^2 - 0\right)$$

$$= \frac{1}{2} \times 10^{-3} \times 50 \times 50$$

$$= 1.25 \text{ J}$$

Assuming that g is (nearly) a constant (with a value$\simeq$ 10 m/s²), the work done by the gravitational force is,

$$W_g = mgh = 10^{-3} \times 10 \times 10^{3} \text{J} = 10.0 \text{ J}$$

From the work-energy theorem

$$\Delta K = W_g + W_r$$

where W_r is the work done by the resistive forces on the raindrop. Thus,

$$W_r = \Delta K - W_g = (1.25 - 10) \text{ J} = -8.75 \text{ J}$$

The negative sign, for this work, implies that the resistive forces act in a direction opposite to the direction in which the ball is falling.

A.2.3.1 (B)

We have, for the first case, $2^2 = u^2 - 2 \times 10 \times 3;\ u^2 = 64$

And for second case , $0 = u^2 - 2 \times 10 \times 3;\ u_0^2 = 60$

The work done in this case is the imparted *KE.* So, their ratio is equal to the ratio of the squares of the initial speed.

So, if the work done in the first case and second case are W and W' respectively , then, we have, $\frac{W}{W'} = \frac{60}{64} = \frac{15}{16}$. So, fraction saved $= 1 - \frac{15}{16} = \frac{1}{16} \equiv \frac{100}{16}\% = \frac{25}{4}\% = 6\frac{1}{4}\% = 6.25\%$

So, correct Option is (B)

A.2.3.2 (D)

We know that, for a spring, stretched by an amount *x*,

$W = \frac{1}{2}kx^2$

(i) in case (a), we would have

$x_{1a} = \frac{F}{k_1}$ and $x_{2a} = \frac{F}{k_2}$

$\therefore W_{1a} = \frac{1}{2}\frac{F^2}{k_1}$ and $W_{2a} = \frac{1}{2}\frac{F^2}{k_2}$

Since $k_1 < k_2$, we have $W_{1a} > W_{2a}$

(ii) In case (b), we would have

$F_{1b} = k_1 x$ and $F_{2b} = k_2 x$

$\therefore\ W_{1b} = \frac{1}{2}k_1x^2$ and $W_{2b} = \frac{1}{2}k_2x^2$

Since $k_1 < k_2$, we would have $W_{1b} < W_{2b}$

A.2.3.3 (B)

When the mass *m* is at a depth *x* below the surface of the earth, the gravitational force on it is due only to the inner sphere of radius (*R*-*x*). Hence,

$$F(x) = \frac{G.\frac{4\pi}{3}(R-x)^3\rho m}{(R-x)^2} = \frac{4\pi}{3}G\rho(R - x)m$$

The infinitesimal work done, in moving it a distance d*x* away from the center, towards the surface of the earth, is

$$dW = F(x)dx = \frac{4\pi}{3}\ G\rho m(R - x)dx$$

Hence total work done in moving the mass from the centre, to the surface of the earth, is

$$\mathrm{W} = \frac{4\pi}{3}G\rho m\int_{x=0}^{x=R}(R - x)dx = \frac{4\pi}{3}G\rho m\left|Rx - \frac{x^2}{2}\right|_0^R = \frac{4\pi}{3}G\rho m\left|R^2 - \frac{R^2}{2}\right| = \frac{4\pi}{3}G\rho m\frac{R^2}{2}$$

But $\rho = \frac{M}{\frac{4}{3}\pi R^3}\ \therefore W = \frac{GMm}{2R} = \frac{m}{2}\ \frac{GM}{R^2}R = \frac{mgR}{2}$

A.2.3.4 (B)

When a conservative force does work on a particle the PE of the particle depends only on the distance (*r*) between the point source of the force and the particle.

We are given here that the given conservative force is doing a positive work when the particle is moving towards the origin of the source of the force. Positive work implies that the displacement of the particle is along the direction of the force on it due to the source. The particle, in this case, must, therefore, be experiencing an attractive force due to the 'source'.

By convention, the PE of the particle, under an attractive force, is always taken with a negative sign. It is also proportional to $1/r$ for a conservative inverse square force.

It follows that (because of its negative sign), the PE, in such a case, will decrease in r (It would become more negative). Its maximum value is zero which (because of the inverse proportionality with r) will be attained as

$r \rightarrow \infty$.

A.2.3.5 (D)

The PE, stored in the compressed spring $= \frac{1}{2} k x_0^2$. It is this PE that raises the given object through height H.

Hence

$$\frac{1}{2} k x_0^2 = mgh$$

$$\text{or } k = \frac{2mgh}{x_0^2}$$

A.2.3.6 (C)

The spring constant k, of a spring of a given material, is inversely proportional to the length of the spring and directly proportional to its area of cross section, i.e. d^2.

In case of springs (1) and (2), we would, therefore, have $k_1 < k_2$

and In case of springs (3) and (4), we would, therefore, have $k_3 > k_4$

The PE, stored in a spring, when it is stretched by an amount x, equals $U = \frac{1}{2} k x^2$. Here x being the same in all cases, we have $U \propto k$. Hence we would have

$$U_1 < U_2$$

$$U_3 > U_4$$

[Note: These (approximate) results are obtained by applying Hooke's law to the stretching of a spring. We have

$$\gamma = \frac{4f/\pi d^2}{(\Delta l/L)}$$

Now force per unit change in length = k

$\therefore\ k = Y.\frac{1}{L}\frac{\pi d^2}{4}$ (Here Y is a constant for a given material.)

$\therefore$ For same d, $k \propto \frac{1}{L}$

and for same L, $k \propto d^2$

A.2.3.7 (D) $\int_1^2 F(x)dx$ = Work done is taking the particle from 1 to 2,

= change in KE

= $K_2 - K_1$

If $F(x) = -\frac{dU}{dx}$, then $\int_1^2 F(x)dx = -\int_1^2 dU = U_1 - U_2$

Which will give $K_2 - K_1 = U_1 - U_2$

Or $K_2 + U_2 = K_1 + U_1$, which is consistent with the principle of conservation of energy.

So, Case I is a possibility. Case II will lead to

$K_2 - U_2 = K_1 - U_1$ Which does not conform to the principle of conservation of energy.

A.2.3.8 (D)

For path 1

$$W_1 = [q(c_1\hat{i} + c_2\hat{j})].\left[\{(1-0)\,\hat{i} + (2-0)\,\hat{j}\} + \{(4-1)\,\hat{i} + (5-2)\,\hat{j}\}\right] = q[4C_1 + 5C_2]$$

For path 2

$$W_2 = [q(C_1\hat{i} + C_2\hat{j})].[(4-0)\,\hat{i} + (5-0)\,\hat{j}] = q(4C_1 + 5C_2)$$

For path 3

$$W_3 = [q(C_1\hat{i} + C_2\hat{j})].[\{(2-0)\,\hat{i} + (1-0)\,\hat{j}\} + \{(4-2)\,\hat{i} + (5-1)\,\hat{j}\}] = q[4C_1 + 5C_2]$$

Thus $W_1 = W_2 = W_3$. Hence the electric force is a conservative force.

A.2.3.9 (D)

Over a complete orbit, the final position (and final speed) of the comet is the same as its initial position. Hence, there is no change in the PE, or the KE, of the comet. This implies that the net work done on the comet, by the gravitational force of the Sun, (over a complete orbit) is zero.

A.2.3.10 (C)

We have,

Centripetal force = $\left(\frac{mv^2}{r}\right) = \frac{e^2}{4\pi\epsilon_0 r^2}$

$$\therefore mv^2 r = \frac{1}{4\pi\epsilon_0}e^2$$

or $$\left(\frac{1}{2}mv^2\right)r = \frac{1}{2}.\frac{e^2}{4\pi\epsilon_0}$$

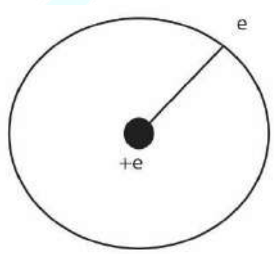


Fig.A.2.3.1

This equation appears to indicate that as the electron radiates energy, its kinetic energy $\left(\frac{1}{2}mv^2\right)$should decrease and, therefore, r would increase (Fig. A.2.3.1).

However, we need to remember that the electron, moving around the nucleus, also has a PE $\left(= -\frac{1}{4\pi \epsilon_0}\frac{e^2}{r} = -mv^2 = -2KE\right)$associated with it. As the electron radiates energy, its total energy (and not its KE) decreases. The PE (because of its negative sign) decreases twice as fast as the KE when r decreases.

Hence, the total energy can decrease even when the KE increases.

The electron, therefore continuously moves towards the nucleus with a continuous increase in its KE {to keep $\left(\frac{1}{2}mv^2\right)r$ = constant}.

There is, however, a continuous decrease in its total energy.

Module 2.4 Elastic and Inelastic Collision

A.2.4.1(A) The component of linear momentum of the gas molecule before collision along AB and CD are respectively $mv \sin \theta$ and $mv \cos \theta$ (Fig.A.2.4.1).

And the component along BA and CD are respectively $mv \sin \theta$ and $mv \cos \theta$.

For calculating the force, we need to determine the change in momentum.

For determining the change, we find that equal components along the CD neutralize each other.

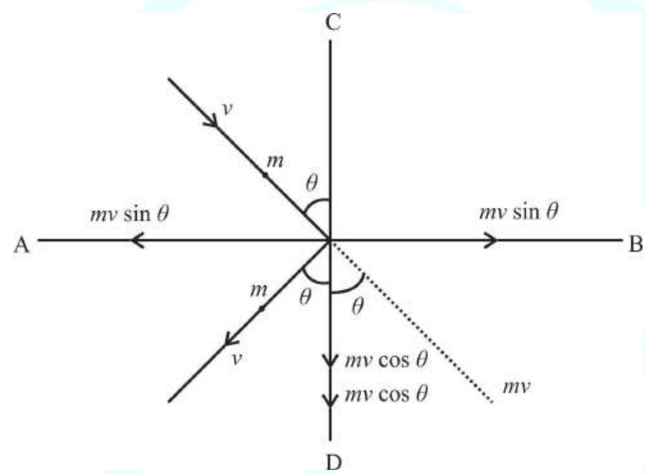


Fig.A.2.4.1

We are left with the components along AB and BA.

So the change along BA, i.e. OA is equal to $mv \sin \theta - (-\ mv \sin \theta) = 2mv \sin \theta$

Since, the time of impact of the collision is τ, the force = $\frac{2mv \sin \theta}{\tau}$ along OA.

A.2.4.2 (C)

The velocity, of object (2), at $t = 0$, equals v_0, (the velocity with which object (1), of same mass, collides elastically with it).

At time $t = t_0$, objects (2) and (3) have equal velocities, say, v each. Also, at this instant, the spring has been compressed by an amount x_0.

Hence, by principle of energy conservation,

$$\frac{1}{2}mv_0^{\ 2} = \frac{1}{2}mv^2 + \frac{1}{2}.2mv^2 + \frac{1}{2}kx_0^{\ 2} \qquad (2.4.1)$$

Also, by momentum conservation, we get

$$mv_0 = mv + 2mv \qquad (2.4.2)$$

This gives $v = v_0/3$

Substituting the value of v in Eq(2.4.1), we get

or $\quad kx_0^2 = mv_0^2 - \frac{3mv_0^2}{9} = \frac{2}{3}mv_0^2$

$$\therefore k = \frac{2}{3}\frac{mv_0^2}{x_0^2}$$

A.2.4.3 (B)

At the start, the bob is raised to a height L.

Let the coefficient of restitution corresponding to bob wall collision be e.

When the angular amplitude of the oscillation becomes 60^0, the height of the bob, would be

$$h = L(1 - \cos 60°) = \frac{L}{2}$$

The velocities of the bob, before and after the first collision, are $\sqrt{2gl}$ and (e$\sqrt{2gl}$), respectively. Hence, the height h_1, to which the bob would rise after the first collision, equals e^2L. So, the corresponding height, after two collisions would be e^4L .

The height h_n, to which the bob would rise after n collisions, would, in a similar way, be given by

$$h_n = e^{2n}L$$

We want the value of n for which $h_n = \frac{L}{2}$

$$\therefore (e)^{2n}. L = \frac{L}{2}, \qquad \therefore (e)^{2n} = \frac{1}{2}$$

$$\therefore \left(\frac{2}{\sqrt{5}}\right)^{2n} = \frac{1}{2}$$

Taking logarithm to the base 10 on both sides, we get

$$2n(\log\ 2\ \ -\ \log\ \sqrt{5}\ ) = (\ -\ \log\ 2\ )$$

$$2n[0.3\ -\ 0.35] = (-\ 0.3)$$

$$\therefore\ \ 2n = \frac{0.3}{0.05} = 6$$

$$\text{or}\ \ n = 3$$

A.2.4.4 (A)

The ball would take a time t_1, to make its first impact with the floor, where

$$H = 0 + \frac{1}{2}gt_1^2$$

$$\therefore t_1 = \sqrt{\frac{2H}{g}}$$

The ball would rebound with a velocity $e\sqrt{2gH}$. Hence the time, t_2 taken by it to rise to its new (reduced) height, say, h_1, is given by

$$0 = e\sqrt{2gH} - gt_2$$

$$\text{or}\ \ t_2 = \frac{e\sqrt{2gH}}{g} = e\sqrt{\frac{2H}{g}}$$

The ball would again take a further time t_2, before it makes its second impact with the floor. Hence the time instant, at which it would make its second impact with the floor, is

$$t_1 + t_2 + t_2 = t_1 + 2t_2$$

$$= \sqrt{\frac{2H}{g}} + 2e\sqrt{\frac{2H}{g}} = \sqrt{\frac{2H}{g}}(1 + 2e)$$

A.2.4.5 (C)

The collision being elastic, we have

$$m\vec{v} = m\vec{v}_1 + m\vec{v}_2$$

$$\therefore \vec{v} = \vec{v}_1 + \vec{v}_2 \qquad (2.4.3)$$

and $\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

$$\therefore v^2 = v_1^2 + v_2^2 \qquad (2.4.4)$$

From the Eq (2.4.3), we have

$$\vec{v}.\vec{v} = \left(\vec{v}_1 + \vec{v}_2\right).\left(\vec{v}_1 + \vec{v}_2\right)$$

$$v^2 = v_1^2 + 2\vec{v}_1.\vec{v}_2 + v_2^2$$

Using the Eq (2.4.4), we get

$$v_1^2 + v_2^2 = v_1^2 + v_2^2 + 2\vec{v}_1.\vec{v}_2$$

$$\therefore \vec{v}_1.\vec{v}_2 = 0$$

Or $v_1 v_2 \cos(\theta_1 + \theta_2) = 0$

$$\therefore \theta_1 + \theta_2 = \frac{\pi}{2}$$

$$\therefore \theta_2 = \frac{\pi}{3}$$

A.2.4.6 (A)

The height of rise, after the first impact is $e^2 H$. Hence,

$162 = e^2 H = e^2.\ 200$

$$\therefore e^2 = \frac{162}{200} = 0.81$$

$$\therefore e = 0.9$$

The height of rise after the second impact would be $e^2 \times 162$ cm

Hence, this height is

$0.81 \times 162 = 131.22$ cm or 131.00 cm (nearly)

A.2.4.7 (B)

The velocity v_1, with which the ball hits the floor, just before the first impact, is

$$v_1 = \sqrt{2gH}$$

Hence, its velocity of rebound, is

$$v_2 = ev_1 = e(\sqrt{2gH})$$

The height, h_1, to which it would rise, would be given by

$$h_1 = \frac{v_2^2}{2g} = e^2 H$$

At the second impact, the ball would hit the floor with a velocity v_2 and would rebound with a velocity ev_2

Hence the height, h_2, to which it would rise, after to record impact, is

$$h_2 = \frac{(ev_2)^2}{2g} = e^4 H$$

Continuing, in the same way, the height h_3, to which it would rise, after the third impact, is

$$h_3 = e^6 H$$

A.2.4.8 (A)

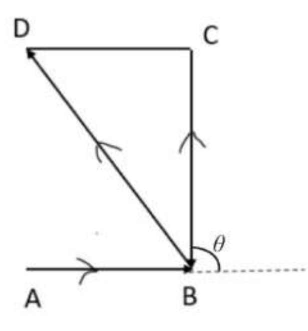


Fig A.2.4.2

Let $\vec{AB}$ represent the original momentum vector in magnitude and direction. $\vec{CD}$ represents the same after impact (Fig A.2.4.2).

$$\vec{\Delta p} = \vec{BC} - \vec{AB} = \vec{BC} + \vec{CD} = \vec{BD}$$

$$\left|\vec{\Delta p}\right| = \sqrt{BC^2 + CD^2} = \sqrt{BC^2 + AB^2}$$

Initial speed = 144 km/h = 40 m/s ;

Final speed = 270 km/h = 75 m/s

Initial momentum $AB = 0.16 \times 40 = 6.4$ kg m/s

Final momentum $BC = 0.16 \times 75 = 12$ kg m/s

$\therefore$ change in momentum= $\left|\vec{\Delta p}\right| = \sqrt{(6.4)^2 + 12^2}$

$= \sqrt{40.96 + 144}$

$= \sqrt{184.96} = 13.6$ kg m/s

$= \frac{CD}{BC} = \frac{6.4}{12} = \frac{8}{15}$; $\theta = \frac{\pi}{2} + \tan^{-1}\frac{CD}{BC}$

$= \frac{\pi}{2} + \tan^{-1}\frac{8}{15}$, $\theta = \tan^{-1}\frac{8}{15} + \frac{\pi}{2}$

A.2.4.9 (C)

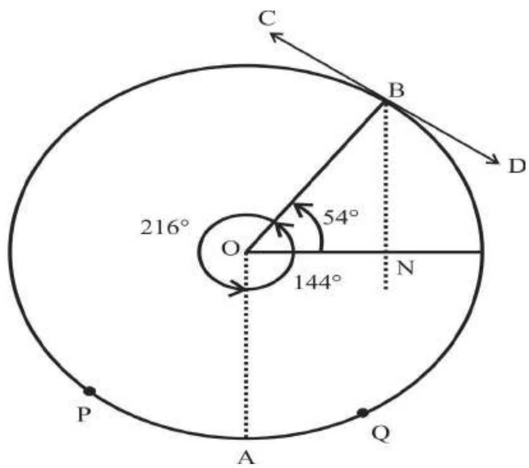


Fig.A.2.4.3

The angular displacements of P and Q at the instant of collision is given by,

$\frac{3}{3+2} \times 360 = 216,$ $\frac{2}{3+2} \times 360 = 144$

So, P and Q meet at a point B such that the greater among the angles AOB is 216° and the smaller is 144°. BN is the perpendicular from B on the radial direction perpendicular to OA. So angle BON is equal to 54°.

$\therefore$ ∠NBD= 54° (Fig.A.2.4.3).

The momenta of P and Q at the time of collision are given by $3mu$ and $2mu$ along the direction of BD and BC respectively. So, from the principle of conservation of momentum

$$3mu - 2mu = mv - 0$$

$$\therefore u = v$$

Hence the final momentum of particle Q will be mu along a direction perpendicular to the radius at B.

The component of final momentum of particle Q will therefore be given by

$mu \cos 54° = mu \sin 36°$.

A.2.4.10 (C)

Initial KE $= \frac{1}{2} m\, u^2$

From the law of conservation of angular momentum

$$mu = (m + M)v$$

$$v = \frac{mu}{m+M}$$

So, decrease in KE

$$= \frac{1}{2} mu^2 - \frac{1}{2}(m + M)\left(\frac{mu}{m+M}\right)^2 = \frac{1}{2} mu^2\left(1 - \frac{m}{m+M}\right) = \frac{1}{2}\left(\frac{Mm}{M+m}\right)u^2 = \frac{1}{2} m_0 w^2$$

So, $m_0 = \left(\frac{Mm}{M+m}\right)$ and $w = u$

$$m_0 < m$$

So the answer is (C).

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3. System of Particles and Rotational Motion

Module 3.1 System of Particles

Q.3.1.1 Given below are three statements (p), (q) and (r) in respect of the reduced mass (μ) of the two body electron- proton system, having individual masses m_e, m_p respectively.

(p) $m_e < \mu < m_p$

(q) $m_e < m_p < \mu$

(r) $\mu < m_e < m_p$

Identify the correct statement among the following.

(A) only (p)

(B) only (q)

(C) only (r)

(D) It may vary between (p) and (q)

Q.3.1.2 Given below are four statements (p), (q), (r), and (s) about the reduced mass μ of the Sun-earth system –

(p) μ is less than each of the masses of the Sun and the earth

(q) μ is greater than each of the masses of the Sun and the earth

(r) μ is equal to the arithmetic mean of the masses of the Sun and the earth

(s) μ is equal to the harmonic mean of the masses of the Sun and the earth

Which among the following statements is/are correct?

(A) only (p)

(B) only (q)

(C) (q) and (r)

(D) (p) and (s)

Q.3.1.3 Which one among the following statements is/are correct in respect of the two-body system under the influence of their mutual action and reaction forces?

(p) The centre of mass of the system drifts in space with a uniform velocity.

(q) A frame of reference can exist with respect to which the centre of mass remains at rest.

(r) The two equations representing the motion of two bodies can be reduced to a single equation in terms of the relative coordinates of the two bodies.

(A) only (p)

(B) only (q)

(C) All three, (p), (q) and (r)

(D) (p) and (q), but not (r)

Q.3.1.4 A square hole is cut out from a uniform circular lamina of radius *a* as shown in the Fig.Q.3.1.1. The distance of the centre of gravity of the remaining lamina from the centre of the circle is

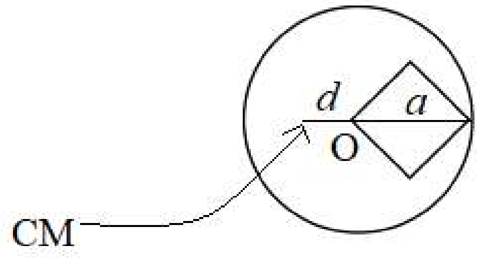

Fig.Q.3.1.1

(A) $\frac{a}{2\pi}$

(B) $\frac{a}{2\pi-1}$

(C) $\frac{a}{4\pi}$

(D) $\frac{a}{4\pi-2}$

Q.3.1.5 The centre of mass of a triangular lamina is located at its

(A) circumcentre

(B) in centre

(C) centroid

(D) ortho centre

Q.3.1.6 Find the coordinates of the centre of mass of the portion of the first quadrant of the circle $x^2 + y^2 = a^2$

(A) $\left(\frac{4a}{3\pi}, \frac{4a}{3\pi}\right)$

(B) $\left(\frac{4a}{3\pi}, \frac{8a}{3\pi}\right)$

(C) $\left(\frac{8a}{3\pi}, \frac{4a}{3\pi}\right)$

(D) $\left(\frac{8a}{3\pi}, \frac{8a}{3\pi}\right)$

Q.3.1.7 For a two-particle system with masses 1 kg and 2 kg, the velocity of the first particle about to the center of mass is 2 ms^{-1} along a straight line AB. The velocity of the second particle about the centre of mass will be

(A) 1 ms^{-1} along AB

(B) 1 ms^{-1} along BA

(C) 2 ms^{-1} along AB

(D) 2 ms^{-1} along BA

Q.3.1.8 For a two-particle having masses 1 kg and 2 kg, the kinetic energy about its centre of mass is 15 J and the velocity of the centre of mass is 2 ms^{-1}. The total kinetic energy of the system is

(A) 9 J

(B) 10 J

(C) 15 J

(D) 21 J

Q.3.1.9 A bomb is thrown and it explodes while moving in its trajectory. The centre of mass of the fragments after explosion will follow a/an

(A) parabolic path

(B) elliptical path

(C) linear path

(D) hyperbolic path

Q.3.1.10 Given below are four statements, pertaining to the centre of mass coordinates and the reduced mass of a two-body system. In which case the mass of one body is much larger than that of the other?

(A) The centre of mass coordinates are very nearly the same as that of the heavier mass and the reduced mass is approximately equal to the heavier mass.

(B) The centre of mass coordinates are almost the same as that of the lighter mass and the reduced mass is approximately equal to the heavier mass.

(C) The centre of mass coordinates are almost the same as that of the heavier mass and the reduced mass is nearly equal to the lighter mass.

(D) The centre of mass coordinates are almost the same as that of the lighter mass and the reduced mass is nearly equal to the lighter mass.

Q.3.2.1 If ω_1, ω_2 and ω_3 are the angular speeds of the seconds hand, minute hand and hour hand respectively of a watch then $\omega_1:\omega_2:\omega_3$ is equal to

(A) 360 : 6 : 1

(B) 360 : 12 : 1

(C) 720 : 10 : 1

(D) 720 : 12 : 1

Q.3.2.2 The radius of the dial of a wrist watch is 2 cm. The angular velocity of the midpoint of the minute hand in radians per second is

(A) $\frac{\pi}{900}$

(B) $\frac{\pi}{1200}$

(C) $\frac{\pi}{1800}$

(D) $\frac{\pi}{3600}$

Q.3.2.3 Statements (p), (q), (r), and (s) pertain to the rotation of a rigid body about a fixed axis

(p) The magnitude of the angular velocity vector does not change with time.

(q) The magnitude of the angular velocity vector may change with time.

(r) The direction of the angular velocity vector does not change with time.

(s) The direction of the angular velocity vector may change with time.

Which among the above statements are correct?

(A) (p) and (q)

(B) (p) and (r)

(C) (q) and (r)

(D) (q) and (s)

Q.3.2.4 When observed at time $t_0 = 0$, a ceiling fan has angular velocity ω_o. After a time t it describes an angle θ and attains angular velocity ω. If the angular velocity of fan is assumed constant and its angular acceleration is α, then ω can be expressed by which of the following expression or expressions?

(p) $\frac{2\theta}{t} - \omega_o$

(q) $\omega_o + \alpha t$

(r) $\omega_o^{\,2} + 2\alpha\theta$

(s) $\left(\omega_o^{\,2} + \frac{1}{2}\alpha\theta\right)^{\frac{1}{2}}$

(A) Only (p)

(B) (p) and (q)

(C) Only (r)

(D) Only (s)

Q.3.2.5 For a particle executing uniform circular motion, the angular speed is 10 rad s^{-1} . The angular speed of the particle about a point on the circumference of the circle is

(A) 5 rad s^{-1}

(B) 10 rad s^{-1}

(C) 20 rad s^{-1}

(D) dependent on the location on the circumference of the point concerned

Q.3.2.6 A body rotating about an axis has an initial angular velocity, ω_o and angular acceleration α,. The angle described by it during the tenth second is

(A) $\omega_o + (9.5)\alpha$

(B) $\omega_o + 4\alpha$

(C) $5(\omega_o + \alpha)$

(D) $5\omega_o + 4\alpha$

Stem for Question Q.3.2.7 and Q.3.2.9

A body executing uniformly accelerated circular motion describes 8π rad during the first 2 seconds and 30π rad in the first 6 seconds.

Q.3.2.7 What is the value of its angular acceleration?

(A) $\frac{\pi}{6}$ rad/s^2

(B) $\frac{\pi}{4}$ rad/s^2

(C) $\frac{\pi}{3}$ rad/s^2

(D) $\frac{\pi}{2}$ rad/s^2

Q.3.2.8 What is the number of revolutions undergone by the body in 10 seconds?

(A) 10

(B) 20

(C) 30

(D) 40

Q.3.2.9 What is the value of angular velocity of the body attained after 10 seconds?

(A) 7π

(B) 8.5π

(C) 9.5π

(D) 12π

Q.3.2.10 A body is moving with a uniform velocity u along the straight line $x = a$. What is the value of its angular velocity about the origin when its end makes an angle 45° with the x-axis?

(A) $\frac{u}{a}$

(B) $\frac{u}{2a}$

(C) $\frac{2u}{a}$

(D) $\frac{\sqrt{2u}}{a}$

Module 3.3 Dynamics of Rotational Motion

Q.3.3.1 Given below are three statements about the moment of a couple

(p) Its value does not depend on the location of the fulcrum

(q) Its value depends on the location of the fulcrum

(r) Its value becomes zero if the fulcrum point lies on the line of action of any of the forces constituting the couple

Which among the following is/are true?

(A) Only (p)

(B) Only (q)

(C) Only (r)

(D) (q) and (r)

Q.3.3.2 P and Q are two points on the lines of action of two equal and opposite parallel forces that make an angle θ with PQ. At this stage the moment of the couple is A (Fig. Q.3.3.1). The lines of action of the forces are now turned through 90°.The moment of the couple now is B. The moment of the couple when the lines of action of the forces are perpendicular to PQ is

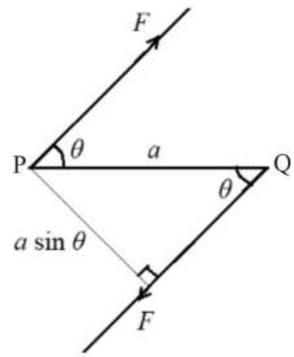

Fig. Q.3.3.1

(A) $A+B$

(B) $A-B$

(C) $\sqrt{A^2+B^2}$

(D) $\sqrt{A^2-B^2}$

Q.3.3.3 A circular plate of radius R has a mass, M. Its moment of inertia about one of its diameter is

(A) MR^2

(B) $\frac{1}{4}MR^2$

(C) $\frac{1}{3}MR^2$

(D) $\frac{1}{2}MR^2$

Stem for Question Q.3.3.4 and Q.3.3.5

A merry-go-round in the form of a heavy wooden disc of mass M and radius R is being pushed at its edge continuously by a person by a force of magnitude F. Friction at the mechanism used for turning is negligible.

Q.3.3.4 What is the angular acceleration acquired by the merry-go-round ?

(A) $\frac{F}{MR}$

(B) $\frac{F}{2MR}$

(C) $\frac{2F}{MR}$

(D) $\frac{4F}{MR}$

Q.3.3.5 A child of mass m_1 is now made to sit near the edge on a seat of mass m_2. What will be the angular acceleration if the child and the seat can be considered as point masses at the edges.

(A) $\frac{2F}{(M+m_1+m_2)R}$

(B) $\frac{2F}{(M+2m_1+2m_2)R}$

(C) $\frac{F}{(2M+m_1+m_2)R}$

(D) $\frac{2F}{(2M+m_1+m_2)R}$

Q.3.3.6 A body of radius R rolls without slipping down an inclined plane of inclination θ with the horizon with an acceleration $\frac{3}{5}g\sin\theta$. The body is a

(A) hollow cylinder

(B) solid cylinder

(C) hollow sphere

(D) solid sphere

Q.3.3.7 Four bodies (a hollow cylinder, a solid cylinder, a hollow sphere, a solid sphere), each of the same radius, are made to roll without slipping down an inclined plane. If the velocities of the bodies on reaching the base are v_1, v_2, v_3 and v_4 respectively, then

(A) $v_3 > v_2 > v_4 > v_1$

(B) $v_3 > v_4 > v_2 > v_1$

(C) $v_4 > v_3 > v_2 > v_1$

(D) $v_4 > v_2 > v_3 > v_1$

Q.3.3.8 Match the terms in the two columns of the given Table from the point of view of rotational analogues of the linear motion.

Displacement (A)	Torque (P)

Mass (B)	Angular displacement (Q)
Force (C)	Angular momentum (R)
Linear momentum (D)	Moment of inertia (S)

(A) A-Q, B-P, C-S, D-R

(B) A-R, B-S, C-P, D-Q

(C) A-R, B-Q, C-S, D-P

(D) A-Q, B-S, C-P, D-R

Q.3.3.9 A simple pendulum having a bob of mass m and effective length l is released from a position where its angular displacement from the mean position is 60°. The work done by the restoring torque in reducing the angular displacement to 30° is

(A) $-\sqrt{3-1}\ mgl$

(B) $\sqrt{3-1}\ mgl$

(C) $-\frac{1}{2}\ mgl$

(D) $\frac{1}{2}\ mgl$

Q.3.3.10 A young girl stands at the centre of a turntable with her two arms outstretched. The turn-table is set into rotation with an angular speed of 60 rev/min. How much is the angular speed of the girl if she folds her hands back and thereby reduces her moment of inertia to 60% of its initial value? Assume that the turn-table rotates without friction.

(A) $\frac{2\pi}{5}$ rad s^{-1}

(B) $\frac{5\pi}{4}$ rad s^{-1}

(C) $\frac{10\pi}{3}$ rad s^{-1}

(D) $\frac{12\pi}{5}$ rad s^{-1}

ANSWERS

Module 3.1 System of Particles

A.3.1.1 (C)

$\frac{m_e m_p}{m_e + m_p}$ is less than both m_e and m_p

A.3.1.2 (D)

$\mu < m_e < m_p = \frac{M_s M_E}{M_s + M_E}$, $\mu < M_s$, $\mu < M_E$

$\mu < m_e < m_p = (\frac{1}{M_E} \ + \frac{1}{M_s})$, which is the harmonic mean between M_s and M_E.

A.3.1.3 (C)

Motion of the centre of mass is affected only by an external unbalanced force.

A.3.1.4 (D)

Let the length of each side of the square hole be x .

$\therefore x^2 + x^2 = a^2$

Or, $x^2 = \frac{a^2}{2}$

Let $\sigma = \frac{mass}{area}$

$\therefore$ Mass of original lamina = $\pi a^2 \sigma$

$\therefore$ Mass of the remaining lamina

$$= \left(\pi a^2 - \frac{a^2}{2}\right)\sigma = \left(\pi - \frac{1}{2}\right)a^2\sigma$$

The required distance d will be given by:

$$\left(\pi - \frac{1}{2}\right)a^2\sigma d = \frac{a^2}{2}\sigma\frac{a}{2}$$

$$\therefore d = \frac{a}{4\pi - 2}$$

A.3.1.5 (C)

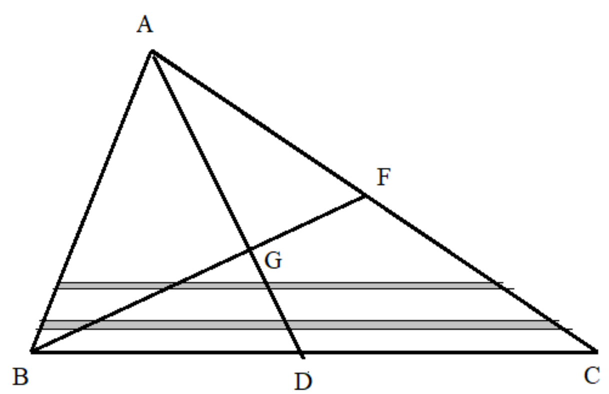

Fig. A.3.1.1

The triangular lamina is cut into multiple infinitesimally thin rectangular strips, each parallel to the base, BC. The centre of mass of each such strip lies on the respective mid-point. So the centre of mass of the triangular lamina which is a combination of such strips lies on the join of the mid-points. So it lies on the median AD. If a similar exercise is done with strips parallel to AC, it will be found that the centre of mass. will be on the median BF. So, the centre of mass will be located at the point of intersection of the medians, that is the centroid.

A. 3.1.6 (B) Refer to Fig.A.3.1.2

The coordinates of the centre of mass are

$$\bar{x} = \frac{\int x \mathrm{d}m}{\int \mathrm{d}m},$$

$$\bar{y} = \frac{\int y \mathrm{d}m}{\int \mathrm{d}m},$$

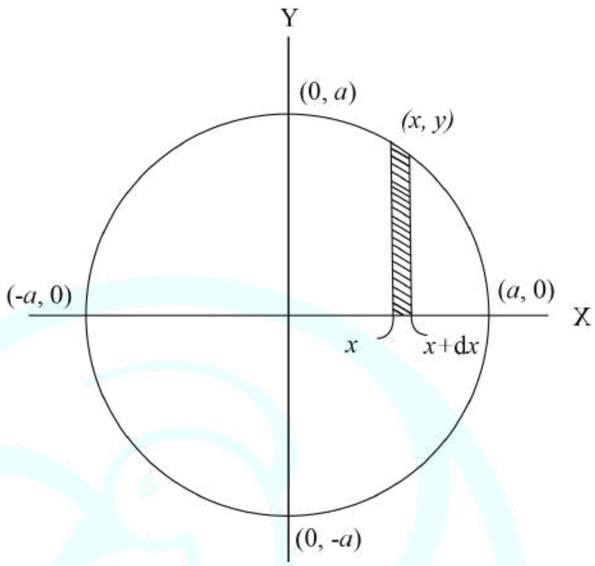

Fig. A.3.1.2

where d*m* is the mass of the elementary strip contained between *x* and *x* + d*x* about the point (*x*,*y*) on the circle. If σ is the mass per unit area of the circular lamina, then d*m* = σ*y*d*x*.

The integrals extend between the limits of the coordinates of the endpoints of the quadrant.

$$\therefore \bar{x} = \frac{\int xy\mathrm{d}x\sigma}{\int y\mathrm{d}x\sigma} = \frac{\int xy\mathrm{d}x}{\int y\mathrm{d}x} = \frac{I_1}{I_2},$$

where $I_1 = \int xy\mathrm{d}x = \int_a^0 x\sqrt{a^2 - x^2}\, \mathrm{d}x$

And $I_2 = \int y\mathrm{d}x = \int_a^0 \sqrt{a^2 - x^2}\, \mathrm{d}x$

$x = a\cos\theta$, $dx = -a\sin\theta d\theta$

$\therefore a^2 - x^2 = a^2 - a^2\cos^2\theta = a^2\sin^2\theta$

$\therefore \sqrt{a^2 - x^2} = a\sin\theta$

When, $x = 0$, $\cos\theta = 0$, $\theta = \frac{\pi}{2}$

$x = a$, $\cos\theta = 1$, $\theta = 0$

$$\therefore I_1 = \int_o^{\pi/2} (a\cos\theta)(a\sin\theta)(-a\sin\theta)d\theta$$

$$= -a^3\int_o^{\pi/2} \sin^2\theta\cos\theta d\theta$$

Let $\sin\theta = u$, so, $+\cos\theta d\theta = du$

When $\theta = 0$, $u = 0$

$\theta = \frac{\pi}{2}$, $u = 1$

$$\therefore I_1 = -a^3\int_o^1 u^2 du = -a^3\left(\frac{1}{3}\right) = -\frac{a^3}{3}$$

$$I_2 = \int_a^0 \sqrt{a^2 - x^2}\, dx = \int_o^{\pi/2} (a\sin\theta)(-a\sin\theta d\theta)$$

$$= -a^2\int_o^{\pi/2} \sin^2\theta d\theta = -a^2\frac{\pi}{4}$$

$$\bar{x} = \frac{I_1}{I_2} = \frac{-\frac{a^3}{3}}{-a^2\frac{\pi}{4}} = \frac{4a}{3\pi}$$

$$\bar{y} = \frac{\int y\sigma y\, \mathrm{d}x}{\int y\, \mathrm{d}x\, \sigma} = \frac{\int y^2 \mathrm{d}x}{\int y \mathrm{d}x} = \frac{J_1}{J_2},$$

Where $J_1 = \int y^2 \mathrm{d}x = \int_a^0 \sqrt{a^2 - x^2}\ \mathrm{d}x$

And $J_2 = \int y \mathrm{d}x = I_2 = -a^2\frac{\pi}{4}$

$$J_1 = -\int_0^a (a^2 - x^2)\, \mathrm{d}x = -(a^2.a - \frac{a^3}{3})$$

$$= -\frac{2a^3}{3}$$

$$\bar{y} = \frac{J_1}{J_2} = \frac{-\frac{2a^3}{3}}{-a^2\frac{\pi}{4}} = \frac{8a}{3\pi}$$

So, the coordinates of C.M. are $(\frac{4a}{3\pi}, \frac{8a}{3\pi})$.

A. 3.1.7 (B)

$$m_1\vec{V}_1' + m_2\vec{V}_2' = 0$$

$$\therefore \vec{V}_2' = -\frac{m_1}{m_2}\vec{v}_1 = -\frac{1}{2}.2\,\hat{AB}\ \mathrm{ms}^{-1} = 1\ \mathrm{ms}^{-1}\text{along BA.}$$

A. 3.1.8 (D)

$$K = K' + \frac{1}{2}Mv^2 = 15 + \frac{1}{2}(1 + 2).2^2 = 15 + 6 = 21\ \mathrm{J}$$

A.3.1.9 (A)

The original path of the bomb is parabolic. Since there is no external force due to the explosion, the centre of mass will continue to follow the original path of the projectile.

A.3.1.10 (C)

$$\boldsymbol{r}_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2},\ \mu = \frac{m_1 + m_2}{m_1 + m_2}$$

Let $m_1 \gg m_2$

$$\boldsymbol{r}_{cm} = \frac{r_1 + \frac{m_2}{m_1} r_2}{1 + \frac{m_2}{m_1}}, \mu = \frac{m_2}{1 + \frac{m_2}{m_1}}$$

As $m_1 >> m_2$, $\frac{m_2}{m_1} \rightarrow 0$

A.3.2.1 (D)

$\omega_1 = \frac{2\pi}{60}$ rad s^{-1}, $\omega_2 = \frac{2\pi}{60\times60}$ rad s^{-1} , $\omega_3 = \frac{2\pi}{12\times60\times60}$ rad s^{-1}

$\therefore\ \omega_1 : \omega_2 : \omega_3 = 720 : 12 : 1$

A.3.2.2 (C) The minute hand rotates with a uniform angular speed equal to $\frac{2\pi}{60\times60}$ rad s^{-1} about an axis perpendicular to the plane of the wrist watch and passing through its centre. Every point on the minute hand has the same angular velocity.

A.3.2.3 (C)

The axis is fixed, so the direction of angular velocity does not change but its magnitude may change. Hence (C) is correct.

A.3.2.4 (B)

$\frac{\omega-\omega_o}{t} = \alpha$ $\quad\therefore\ \omega = \omega_o + \alpha t.$ Hence (q) is correct.

Again, angle described = (Average angular velocity) $\times$ time

$\therefore\ \theta = \frac{\omega-\omega_o}{2} t$, $\therefore \frac{2\theta}{t} = \omega + \omega_o$

$\omega = \frac{2\theta}{t} - \omega_o$ which is (p)

A.3.2.5 (A)

Angle subtended by an arc at any point on the circumference of a circle is half the angle subtended at the centre.

A.3.2.6 (A)

The required angle = $\omega_o.10 + \frac{1}{2}\alpha.10^2 - \omega_o.9 + \frac{1}{2}\alpha 9^2 = \omega_o + \frac{\alpha}{2}(100 - 81) = \omega_o + (9.5)\alpha$

A.3.2.7 (D)

$$\omega_o.2 + \frac{1}{2}\alpha.2^2 = 8\pi$$

$$\omega_o.6 + \frac{1}{2}\alpha.6^2 = 30\pi$$

i.e. $\omega_o + \alpha = 4\pi$

$$\omega_o + 3\alpha = 5\pi$$

$$2\alpha = \pi$$

$$\therefore \alpha = \frac{\pi}{2}$$

A.3.2.8 (C)

$$\omega_o = 4\pi - \frac{\pi}{2} = \frac{7\pi}{2}$$

$\therefore$ Angle described in 10 seconds

$$= \frac{7\pi}{2}.10 + \frac{1}{2}.\frac{\pi}{2}.10^2$$

$= 35\pi + 25\pi = 60\pi$ $\qquad\therefore$ Number of revolutions $= \frac{60\pi}{2\pi} = 30$

A.3.2.9 (B)

Angle described = (Average angular velocity) time

$$\therefore\ 60\,\pi\ = \frac{\omega_o + \omega_1}{2}.\,10\text{ s}$$

$$\therefore 12\,\pi\ = \omega_o + \omega_1 = \frac{7\pi}{2} + \omega_1$$

$$\therefore \omega_1\ = 12\pi\ - \frac{7\pi}{2} = \frac{17\pi}{2} = 8.5\,\pi$$

A.3.2.10 (B)

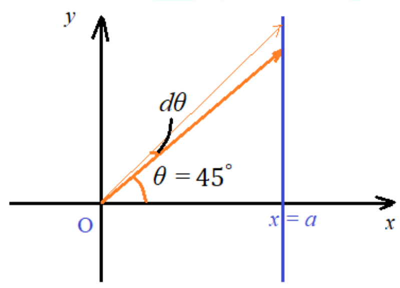

Fig. A.3.2.1

$$\omega\ = \frac{d\theta}{dt} = \frac{1}{1+\frac{y^2}{a^2}}\,\frac{d}{dt}\left(\frac{y}{a}\right)$$

$$\therefore\ \ \omega\ = \frac{a^2}{a^2+y^2}\,\frac{1}{a}\,\frac{dy}{dt}$$

$$= \frac{a}{a^2+a^2\theta}u\quad = \frac{u}{a\theta}\ .$$

Module 3.3 Dynamics of Rotational Motion

A.3.3.1 (A)

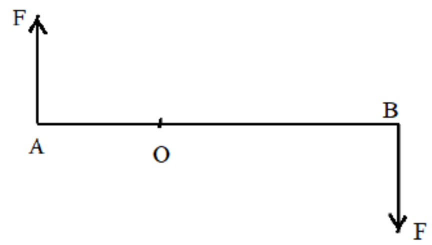

Fig. A.3.3.1

Let O be the point about which the moment of the couple $(\boldsymbol{F}, \boldsymbol{F})$ is being considered (Fig. A.3.3.1).

Moment about O is equal to

$$\boldsymbol{r}_1 \times (-\boldsymbol{F}) + \boldsymbol{r}_2 \times (\boldsymbol{F}) = (-\boldsymbol{r}_2 - \boldsymbol{r}_1) \times \boldsymbol{F} = A\boldsymbol{B} \times \boldsymbol{F},$$

that is independent of the choice of origin O.

A.3.3.2 (C)

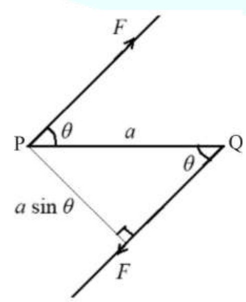

Fig. A.3.3.2

$$A = Fa\sin\theta$$

$$B = Fa\sin(90^{\circ} - \theta) = Fa\cos\theta$$

$$\therefore A^2 + B^2 = F^2a^2(\theta + \theta)$$

$$\therefore A^2 + B^2 = (Fa)^2$$

$$\text{or, } Fa = \sqrt{A^2 + B^2}$$

A.3.3.3 (B)

Moment of inertia about each diameter is the same on account of symmetry. Let us take two perpendicular diameters, and let the corresponding $M\,I$ be I_1 and I_2 respectively.

Then,

$$I_1 = I_2$$

And, from the perpendicular axis theorem,

$$I_1 + I_2 = \frac{1}{2}MR^2$$

$$\therefore I_1 = I_2 = \frac{1}{2}.\frac{1}{2}MR^2$$

$$= \frac{1}{4}MR^2$$

A.3.3.4 (C)

$$\alpha = \frac{Torque}{MI} = \frac{FR}{\frac{1}{2}MR^2} = \frac{2F}{MR}$$

A.3.3.5 (B)

Moment of Inertia of the child and the seat about the axis of rotation $= (m_1 + m_2)R^2$

$$\therefore \alpha = \frac{FR}{\frac{1}{2}MR^2+(m_1+m_2)R^2} = \frac{2F}{(M+2m_1+2m_2)R}$$

A.3.3.6 (C)

$$a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$$

$$\therefore 1 + \frac{k^2}{R^2} = \frac{5}{3}$$

or $\frac{k^2}{R^2} = \frac{2}{3}$

A.3.3.7 (D)

$$v^2 = \frac{2g\sin\theta .s}{1+\frac{k^2}{r^2}}$$

The numerator is the same for all.

For a hollow cylinder, a solid cylinder, a hollow sphere, a solid sphere, the value of $\frac{1}{1+\frac{k^2}{r^2}}$ are

$\frac{1}{2}(= 0.5), \frac{2}{3}(= 0.67), \frac{3}{5}(= 0.6), \frac{5}{7}(= 0.71)$ respectively.

$\therefore v_4 > v_2 > v_3 > v_1$

A.3.3.8 (D)

A.3.3.9 (A)

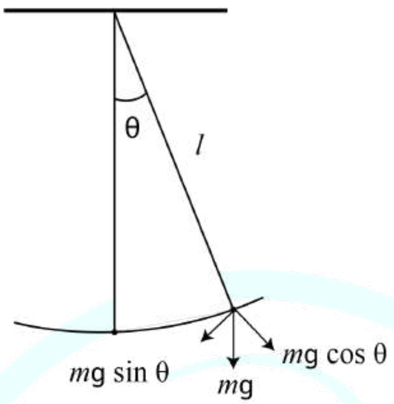

Fig. A.3.3.3

Refer Fig. A.3.3.3

The restoring torque = $mg\sin\theta\, l$

For a displacement, $d\theta$, the work done = $mgl\sin\theta d\theta$

$\therefore$ Work done for angular displacement 30^0 to 60^0

$$= \int_{60^\circ}^{30^0} mgl\sin\theta\, d\theta = -\, mgl|\cos\theta\,|_{60^0}^{30^0} = -\, mgl(\cos 30^0 - \cos 60^0) = \frac{-mgl}{2}\sqrt{3 - 1} = -\,(0.366)\, mgl$$

A.3.3.10 (C)

$$I_1\omega_1 = I_2\omega_2$$

$$\text{or } \omega_2 = \frac{I_1\omega_1}{I_2}$$

$$I_2 = 0.6\, I_1$$

$\omega_1 = 60$ rev/min = 1 rev/s $= 2\pi$ rad/s

$\therefore\ \omega_2 = \frac{I_1 \times 2\pi}{0.6\, I_1} = \frac{10\pi}{3}$ rad/s

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4. Gravitation

Module 4.1 Kepler's Laws and Universal Gravitation

Q.4.1.1 Particles of M, m and $2M$ are masses placed respectively at points A, B and C, where AB = ½ (BC). $m << M$. At time $t = 0$, they are all at rest (Fig.Q.4.1.1).

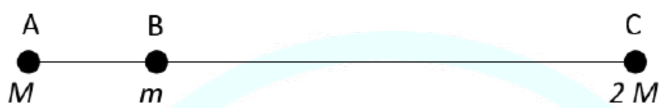

Fig.Q.4.1.1

At subsequent times before any collision takes place, which of the following Options is correct ?

(A) m will remain at rest.

(B) m will move towards M.

(C) m will move towards $2M$.

(D) m will have an oscillatory motion.

Q.4.1.2 Gravitational force between two masses held at a distance r in air is F. When the two masses are inside a liquid with same separation, the gravitational force between them is F'. State which of the following Options is correct?

(A) $F'F$ >1

(B) F'/F $= 1$

(C) F'/F < 1

(D) F'/F will depend on the density of liquid.

Q.4.1.3 Two identical heavy objects, each of mass M are located at P and Q, where PQ $= 2b$. An object of mass $m << M$ is placed at the midpoint of PQ. Then, the object of mass m is displaced by an infinitesimal distance a ($a << b$, such that $(\frac{a}{b})^2$ can be neglected) (i) along the line PQ and (ii) perpendicular to PQ.What is the ratio between the resultant forces experienced by m at the positions (i) and (ii) ?

(A) 1:4

(B) 1:2

(C) 2:1

(D) a: b

Q.4.1.4. Which of the following relations for speed(v) of the planet around the Sun for the positions in the diagram is true?(Fig.Q.4.1.2)

(A) $v_A > v_B > v_D$

(B) $v_A > v_D > v_B$

(C) $v_A > v_C > v_P$

(D) $v_A < v_D < v_P$

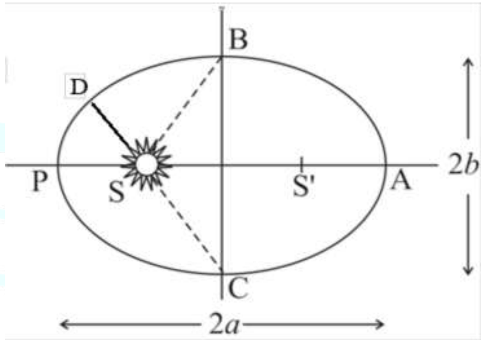

Fig.Q.4.1.2

Q.4.1.5 The areal velocity versus time graph for a planet around the Sun

(A) is a straight line with zero intercept and positive slope.

(B) is a straight line with non-zero intercept and positive slope.

(C) is a straight line with zero intercept and negative slope.

(D) is a straight line with non- zero intercept and negative slope.

Q.4.1.6 A satellite is in a circular orbit around the earth. Due to some reason it loses some of its energy. What is the new trajectory?

(A) Follows a hyperbolic path and falls to earth.

(B) Follows a parabolic path and falls to earth.

(C) Follows a new circular orbit of smaller radius.

(D) Follows an elliptical orbit around earth with earth at one of the two focal points.

Q.4.1.7 A planet moves around the Sun. At point A in its orbit, it is closest to the Sun at a distance d_1 and has a speed v_1. At another point B, when it is farthest from the Sun at a distance d_2 its speed will be

(A) $\frac{d_1 v_1}{d_2}$

(B) $\frac{d_2 v_1}{d_1}$

(C) $\frac{d_1^2 v_1}{d_2^2}$

(D) $\frac{d_2^2 v_1}{d_1^2}$

Q,4.1.8 Mass M is divided into two parts xM and $(1 - x) M$. For a given separation, the value of x for which the gravitational attraction between the two pieces becomes maximum is

(A) 1/3

(B) 1/2

(C) 3/4

(D) 1

Q.4.1.9 A planet is revolving around the Sun in an elliptical path as shown in (Fig.Q.4.1.3).

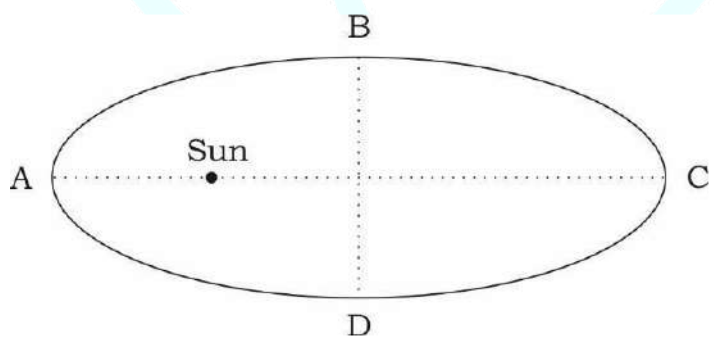

Fig.Q.4.1.3

(A) The time in traveling DAB is less than that for BCD.

(B) The time in traveling DAB is greater than that for BCD.

(C) The time in traveling DAB is equal to that for BCD.

(D) The time in traveling CDA is greater than that for ABC.

Q.4.1.10 A satellite is revolving in an elliptical orbit around the earth (Fig.Q.4.1.4). In order to make its orbit circular, we must fire a rocket

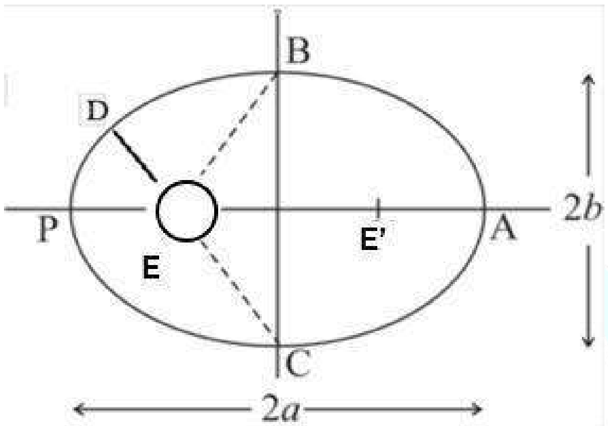

Fig.Q.4.1.4

(A) at aphelion to increase its speed.

(B) at aphelion to decrease its speed.

(C) at perihelion to increase its speed.

(D) at B so as to bring it to rest.

Module 4.2 Gravitational Field

Q.4.2.1 When an extended object is placed below the surface of earth, its center of mass will be

(A) above its center of gravity.

(B) below center of gravity.

(C) coinciding with the center of gravity.

(D) in the same horizontal line as the center of gravity.

Q.4.2.2 The time period of an artificial satellite orbiting around earth is proportional to the nth power of its orbital radius. What is the value of n?

(A) 0.5

(B) -1

(C) 1.5

(D) -2

Q.4.2.3 If the value of g at a height h above the surface of the earth is the same as at a depth d below it (both d and h being much smaller than the radius of the earth), then

(A) $d = h/2$

(B) $d = h$

(C) $d = 2h$

(D) $d = h^2$

Q.4.2.4. Composition of the materials of the two planets are the same. Their radii are r_1 and r_2. The ratio of the acceleration due to gravity g_1/g_2 at the surface of the two planets is

(A) $\frac{r_1}{r_2}$

(B) $\frac{r_2}{r_1}$

(C) $\left(\frac{r_1}{r_2}\right)^2$

(D) $\left(\frac{r_2}{r_1}\right)^2$

Q.4.2.5 The escape velocity from the earth surface is v . The escape velocity from the surface of another planet having a radius twice that of the earth and the same mean density will be

(A) v

(B) $\sqrt{2}v$

(C) $2v$

(D) $2\sqrt{2}v$

Q.4.2.6 A satellite is revolving around the Sun in a circular orbit with uniform velocity v . If the gravitational force suddenly disappears, the magnitude of the velocity of the satellite will be

(A) zero

(B) v /2

(C) v_1

(D) $2v$

Q.4.2.7 A satellite of mass m is moving in a circular orbit of radius R, close to the Earth's surface. Then the work required to be done, to move it to a circular orbit of radius $2R$ is (where M is the mass of the Earth) is

(A) $\frac{2GMm}{R}$

(B) $\frac{GMm}{R}$

(C) $\frac{GMm}{2R}$

(D) $\frac{GMm}{4R}$

Q.4.2.8 The escape velocity of a body from the surface of a planet of mean radius R and density ρ is

(A) $R\sqrt{\frac{2\pi G\rho}{3}}$

(B) $2R\sqrt{\frac{2\pi G\rho}{3}}$

(C) $2R\sqrt{\frac{4\pi G\rho}{3}}$

(D) $4R\sqrt{\frac{2\pi G\rho}{3}}$

Q.4.2.9 A body is projected from the earth's surface to infinity. The kinetic energy needed for the purpose is

(A) $\frac{1}{4}mgR$

(B) $\frac{1}{2}mgR$

(C) mgR

(D) $2mgR$

Q.4.2.10 The total energy (KE + PE) of an artificial satellite in a circular orbit around the Earth is E. Then its PE is equal to

(A) $E/2$

(B) $2E$

(C) $-\ 2E$

(D) $-\ E$

Module 4.3 Earth Satellites and Weightlessness

Q.4.3.1 Which of the following is true in the context of motion of a geostationary satellite around the earth?

(A) It moves around the earth's pole in a north-south direction.

(B) It moves around the earth in an east-west direction.

(C) It goes around the earth in a west-east direction.

(D) It moves around the earth’s pole in an east-west direction.

Q.4.3.2 Suppose a planet that goes around the Sun with angular speed equal to one-eighth of that of the earth. What would be its orbital size as compared to that of the earth?

(A) 8 times that of earth

(B) 4 times that of earth

(C) $1/4^{th}$ of that of earth

(D) $1/8^{th}$ of that of earth

Q.4.3.3 A satellite is to be projected for the purpose of communication. Which of the following is not true?

(A) Its time period should be 24 hr.

(B) It should be in an polar orbit.

(C) It should move from west to east.

(D) It should be at the same height above earth’s surface as that of the other communication satellites.

Q.4.3.4 A person with weight W on earth’s surface is inside an elevator falling with acceleration $g/2$. He/She will feel himself/herself

(A) weightless

(B) having a weight equal to $W/2$

(C) having weight equal to W

(D) having weight equal to $3W/2$

Q.4.3.5 An astronaut in a space station orbiting earth experiences weightlessness. Which of the following is not the correct reason?

(A) Earth's gravity at the space station is zero.

(B) Space station is falling towards the earth with acceleration g.

(C) In the frame of reference of the space station the centrifugal force neutralizes the gravitational attraction.

(D) g at the space station is the same as $\omega^2 r$.

Q.4.3.6 A saturn year is n times the earth year. Semi major axes of the earth orbit = r_E. What are the semi-major axes of saturn orbit?

(A) nr_E

(B) $\sqrt{n}r_E$

(C) $n\sqrt{n}r_E$

(D) $\sqrt[3]{n^2}\,r_E$

Q.4.3.7 Both earth and moon are subject to the gravitational force of the Sun. As observed from the Sun, the possibilities for the nature of the orbit of the moon are as under:

(I) elliptical

(II) not strictly elliptical because the total gravitational force on it is not ideally central.

(III) not elliptical, but will essentially be a closed curve.

Which among the above is/are acceptable?

(A) Only I

(B) II and III

(C) I and II

(D) I and III

Q.4.3.8 During a rescue operation following a fire in a building, whose ground floor and staircase got badly damaged, a fire brigade personnel had to jump from the first floor with a heavy box of weight W on his head. Till such time he touched the ground, he experienced on his head a load

(A) less than W

(B) greater than W

(C) equal to W

(D) equal to zero

Q.4.3.9 An astronaut orbiting the earth in a circular orbit satellite above the surface of the earth gently drops an object out of the satellite. The object will

(A) fall vertically down to the earth

(B) move in an orbit around the earth

(C) will move in an irregular way and fall down to the earth

(D) will escape out of earth's gravity

Q.4.3.10 A satellite is revolving around earth in an elliptic orbit. An astronaut in such a satellite would have apparent weight

(A) equal to zero.

(B) equal to a non-zero constant.

(C) equal to a non-zero quantity varying in magnitude only.

(D) equal to a non-zero varying in magnitude as well as direction.

ANSWERS

Module 4.1 Kepler's Laws and Universal Law of Gravitation

A.4.1.1 (B)

The force of attraction experienced by m due to $2M$ at C will be $F_1 = G\frac{2Mm}{(BC)^2}$ at (along BC).

The force experienced by m due to M at A will be $F_2 = G\frac{Mm}{(AB)^2} = G\frac{4Mm}{(BC)^2}$ (along AB).

Thus $F_2 = 2F_1$. Hence, m will move towards A, i.e. M.

A.4.1.2 (B) Gravitational force between two masses is independent of the medium separating masses.

A.4.1.3 (C)

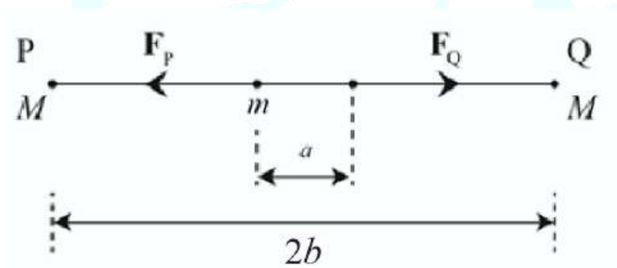

Fig. A.4.1.2

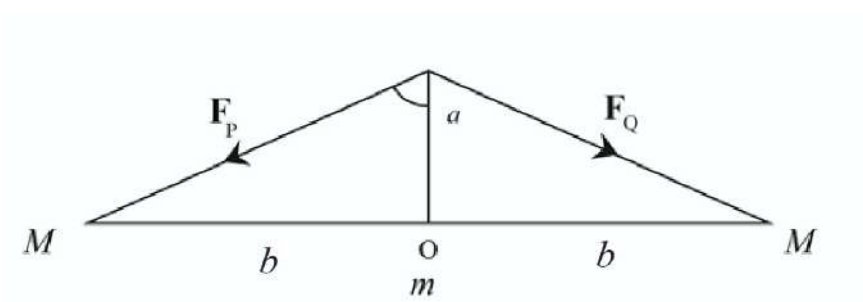

Fig. A.4.1.3

Refer to Fig. A.4.1.2 (for (i))

And Fig A.4.1.3 (for (ii))

The resultant force experienced by m , is given by

$$\mathbf{F}_1 = \mathbf{F}_Q - \mathbf{F}_P$$

$$\therefore \mathbf{F}_1 = GMm\left\{\frac{1}{(b-a)^2} - \frac{1}{(b+a)^2}\right\}$$

$$= GMm\left\{\frac{(b+a)^2-(b-a)^2}{\left(b^2-a^2\right)^2}\right\}$$

$$= GMm\,\frac{4ab}{\left(b^2-a^2\right)^2} = \frac{4\,GMm\,a}{b^2} \times \frac{1}{\left(1-\frac{a^2}{b^2}\right)^2}$$

$$\frac{a^2}{b^2} \to 0, \qquad \text{so, } \mathbf{F}_1 = \frac{4\,GMm\,a}{b^2}$$

In case (ii)

The resultant force experienced by m is given by

$\mathbf{F}_2 = 2\mathbf{F}\cos\theta$, where $\mathbf{F}_p = \mathbf{F}_Q = \mathbf{F}$

$$= \frac{2\,GMm}{\left(a^2+b^2\right)} \cdot \frac{a}{\sqrt{a^2+b^2}} = \frac{2\,GMm\,a}{\left(a^2+b^2\right)^{\frac{3}{2}}}$$

$$\frac{2\,GMm\,a}{b^3\left(1+\frac{a^2}{b^2}\right)^{\frac{3}{2}}} = \frac{2\,GMm\,a}{b^3} \left(\because \frac{a^2}{b^2} \to 0, \right)$$

$$\therefore \mathbf{F}_1 = \mathbf{F}_2 = 4:2 = 2:1$$

A.4.1.4 (D)

The speed of the planet increases from minimum at aphelion (A) to maximum at perihelion (P).

A.4.1.5 (A)

According to Kepler's second law, areal velocity is constant in time.

A.4.1.6 (D)

This is according to Keplar's first law.

A.4.1.7 (A)

As per law of conservation of angular momentum

$$\therefore md_1v_1 = md_2v_2$$

$$v_2 = \frac{d_1v_1}{d_2}.$$

A.4.1.8 (B)

We use the algebraic identity, $ab = \frac{1}{4}\left[(a + b)^2 + (a - b)^2\right]$

So for $(a + b)$ = a constant, ab will be maximum ,when $a = b$. Therefore,in this case $x = (1 - x)$.

Or, $x = ½$.

A.4.1.9 (A)

The planet moves faster when it is closer to the Sun than when it is farther from the Sun to keep areal velocity constant.

A.4.1.10 (A)

At aphelion, the satellite has the lowest speed. In circular orbit we need constant speed. To make orbit circular we may increase speed at aphelion.

Module 4.2 Gravitational Field

A.4.2.1 (A)

The gravitational force acting on the upper part of the object will be greater than the gravitational force acting on the lower most part. Thus the centre of gravity will lie above the centre of mass.

A.4.2.2 (C)

Since $T^2 \propto r^3$

A.4.2.3 (C)

$$g_h = g\left(1 - \frac{2h}{R}\right) = g_d = g\left(1 - \frac{d}{R}\right)$$

$$d = 2h$$

A.4.2.4 (A)

$$g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2} = \frac{4}{3}G\pi\rho R$$

$$\therefore \frac{g_1}{g_2} = \frac{r_1}{r_2}$$

A.4.2.5 (C)

$$v = \sqrt{\frac{2GM}{R}}$$

$$= \sqrt{\frac{2G\frac{4}{3}\pi R^3\rho}{R}}$$

$$= R\sqrt{\frac{8}{3}\pi G\rho}$$

$$\frac{v_P}{v_E} = \frac{R_P}{R_E} = 2$$

A.4.2.6 (C)

If force suddenly disappears, the satellite will continue to move in a straight line with the velocity having the same magnitude along tangential direction with its orbit.

A.4.2.7 (D)

For a satellite in circular orbit of radius r, total energy is, $-\frac{GMm}{2r}$

The work done is the difference in the values of total energy at $r = R$ and $r = 2R$.

The required difference = $-\frac{GMm}{4R} - \left(-\frac{GMm}{2R}\right) = GMm\left(\frac{1}{2R} - \frac{1}{4R}\right) = \frac{GMm}{4R}$

A.4.2.8 (B)

$M = \frac{4\pi}{3}R^3\rho$ in $v_e = \sqrt{\frac{2GM}{R}}$ gives the answer.

A.4.2.9 (C)

Potential energy on the earth's surface is $-\frac{GMm}{R} = -mgR$. Thus kinetic energy needed to send it to infinity is mgR .

A.4.2.10 (B)

With symbol having usual meanings,

$KE = \frac{1}{2}mv^2 = \frac{GMm}{2r}$

$PE = -\frac{GMm}{r}$,

So, $E = -\frac{GMm}{2r}$

$\therefore \quad PE = -2E$

Module 4.3 Earth Satellites and Weightlessness

A.4.3.1 (C)

It goes around the earth in a west-east direction to synchronize with earth's motion because earth rotates from west to east.

A.4.3.2 (B)

4 times that of earth as $r^3\omega^2$ = constant.

A.4.3.3 (B)

Communication needs geostationary equatorial orbit.

A.4.3.4 (B)

Apparent weight = $m(g - a)$.

A.4.3.5 (A)

At the space station $g' = \frac{GM}{r^2} \neq 0$.

A.4.3.6 (D)

$T^2 \propto r^3$

$$\left(\frac{T_S}{T_E}\right)^2 = \left(\frac{r_s}{r_E}\right)^3 n^2 = \frac{r_s^3}{r_E^3} r_s = n^{\frac{2}{3}} r_E$$

A.4.3.7 (B)

Presence of earth alters the central character of the Sun's force. So, the path will not be elliptical, but it will be closed as the moon is bound by the attraction of the earth.

A.4.3.8 (D)

In a freely falling frame, apparent weight is zero.

A.4.3.9 (B)

As the astronaut gently drops the object out, the object continues to have the same speed as a satellite around the earth as before.

A.4.3.10 (D)

With symbols having their usual meanings, in an elliptic orbit mvr = constant but $\frac{mv^2}{r} \neq$ constant. Since the radius vector is not always normal to the ellipse, the direction of the weight keeps changing.

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5. Mechanical Properties of Matter

Module 5.1 Pressure

Q.5.1.1. A vessel is filled with a liquid of density d up to a height h. The pressure exerted on the bottom of the vessel will be the maximum when the vessel is

(A) on the surface of the earth

(B) on top of a hill

(C) deep inside a mine

(D) on the surface of the moon

Q.5.1.2. Two vessels A and B filled with water and glycerine respectively up to a height h exert pressures P_a and P_b at their respective bottoms. Then

(A) $P_a = P_b$

(B) $P_a > P_b$

(C) $P_a < P_b$

(D) the values of P_a and P_b will depend on the cross section of the containers.

Q.5.1.3. A pan(assumed massless) filled with sand is placed on a weighing scale calibrated to read zero. Fig.Q.5.1.1 Marbles, each of mass m, are falling into the pan of unit area at the rate of n per second. The marbles hit the pan with a speed u and stick to the sand without bouncing. At time t second, the pressure exerted at the bottom of the pan is

(A) mu

(B) num

(C) *nmu + n.mtg*

(D) *nmut + n.mg*

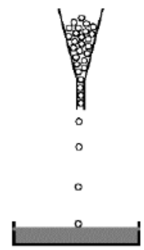


Fig. Q.5.1.1

Q.5.1.4 Which of the following is the SI unit of pressure?

(A) Nm^{-2} only

(B) Jm^{-3} only

(C) kg $m^{-1}s^{-2}$ only

(D) All of the above.

Q.5.1.5 The Fig.Q.5.1.2 shows four points A,B, C and D at different heights from the surface of the earth. P_A, P_B, P_C and P_D denote the atmospheric pressures at the points A, B, C and D respectively. Given

$P_A - P_B = P_B - P_C = P_C - P_D$

Which of the following statements is true?

(A) $h_1 > h_2 > h_3$

(B) $h_1 < h_2 < h_3$

(C) $h_1 = h_2 = h_3$

(D) $h_1 = h_2 > h_3$.

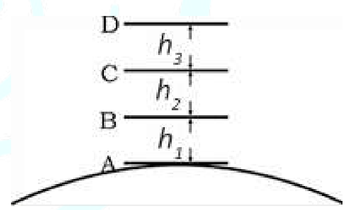


Fig.Q.5.1.2

Q.5.1.6 A, B, C and D are points on the same vertical line near the surface of earth, separated by distances h_1, h_2 and h_3 as shown in Fig.Q.5.1.3. Assume that the density of air remains constant over the points and

$P_A - P_B = P_B - P_C = P_C - P_{D.}$

The correct statement out of the following is

(A) $h_1 > h_2 > h_3$

(B) $h_1 < h_2 < h_3$

(C) $h_1 = h_2 = h_3$

(D) $h_1 = h_2 > h_3$.

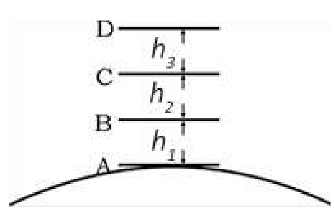


Fig.Q.5.1.3

Q.5.1.7 The atmospheric pressure has the maximum value

(A) inside a deep well.

(B) on the surface of the earth.

(C) 100 km above the surface of the earth.

(D) at a point just above the atmosphere.

Q.5.1.8 The Fig.5.1.4 shows two vessels A and B with varying cross sectional areas. The vessels are filled with water to the same heights. P_A and P_B denote the pressures exerted by water at the bottoms of the vessels. F_A and F_B are the corresponding thrusts. We have

(A) $F_A = F_B; P_A = P_B$

(B) $F_A > F_B; P_A = P_B$

(C) $F_A < F_B; P_A > P_B$

(D) $F_A < F_B; P_A = P_B$

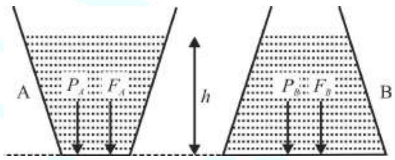


Fig.Q.5.1.4

Q.5.1.9. A tank with a square base is divided by a vertical partition in the middle (Fig.Q.5.1.5). The bottom of the partition has a small hinged door. The two parts of the tank are filled to the same height with different liquids of densities ρ_1 and ρ_2 as shown. Given that $\rho_1 > \rho_2$.

The door experiences

(A) a net force directed towards the right.

(B) a net force directed towards the left

(C) no net force

(D) a net force along the vertical.

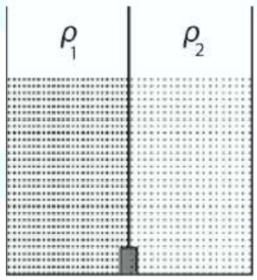


Fig.Q.5.1.5

Q.5.1.10. A manometer reads the pressure of a gas in an enclosure as shown in Fig.Q.5.1.6 (a). When the pump removes some of the gas, the manometer reads as in Fig.Q.5.1.6 (b). The change in pressure in the enclosure expressed in terms of the length of the liquid column is

(A) 2 cm

(B) 38 cm

(C) 40 cm

(D) 36 cm

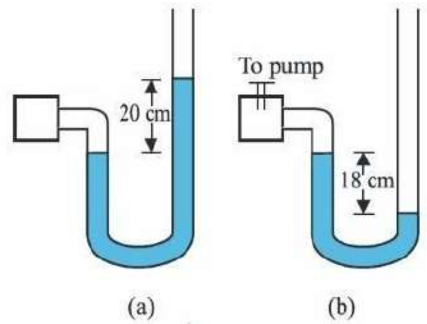


Fig.Q.5.1.6 (a),(b)

Module 5.2 Bernoulli's Principle

Q.5.2.1. The streamlines indicating the flow of an ideal liquid are closely packed in a region. This indicates

(A) an increase in velocity

(B) a decrease in velocity

(C) an increase in density

(D) a decrease in density

Q.5.2.2. A horizontal tube of varying cross-section has a streamlined flow of an ideal liquid. R and $2R$ are the radii of the tube at points P and Q respectively. V_P and V_Q are the speeds of flow of the liquid at P and Q. We have

(A) $V_P = 2V_Q$

(B) $2V_P = V_Q$

(C) $V_P = 4V_Q$

(D) $4V_P = V_Q$

Q.5.2.3. A liquid of density d flows in a tube of radius r. The flow tends to be turbulent if

(A) d decreases, r increases.

(B) d decreases, r decreases

(C) d increases and r increases

(D) d increases and r decreases

Q.5.2.4 Which of the following laws leads to the equation of continuity in fluid flow?

(A) Newton's second law

(B) law of conservation of energy

(C) law of conservation of mass

(D) law of conservation of linear momentum

Q.5.2.5 A horizontal cylindrical tube of radius R open at one end has a plate having N holes each of radius r fixed at the other end. A liquid in streamlined flow enters the open end of the tube with speed v. The speed of liquid emerging out of each hole at the other end is

(A) Rv/Nr

(B) NRv/r.

(C) R^2v/Nr^2

(D) NR^2v/r^2

Q.5.2.6 A thin horizontal plate of area a and mass M is in equilibrium in the gravity of earth without any support due to air(density d) flowing with speed v and nv below and above the plates. The value of M

(A) $(n-1)\, av^2d/2g$

(B) $(n^2-1)\, av^2d/2g$

(C) $(n^2-1)\, av^2d/g$

(D) $(n^2+1)\, av^2d/2g$

Q.5.2.7.During storms, the thatched roofs sometime get blown off because

(A) pressure outside is greater than the pressure inside the room.

(B) pressure inside is greater than the pressure outside the room.

(C) of low speed and low pressure inside.

(D) of high speed and high pressure inside.

Q.5.2.8 Fig.Q.5.2.1 (a), (b), (c), and (d) refer to the steady flow of a non-viscous liquid. The correct Figs. are

(A) (a) and (d)

(B) (b) and (d)

(C) (a) and (c)

(D) (b) and (c)

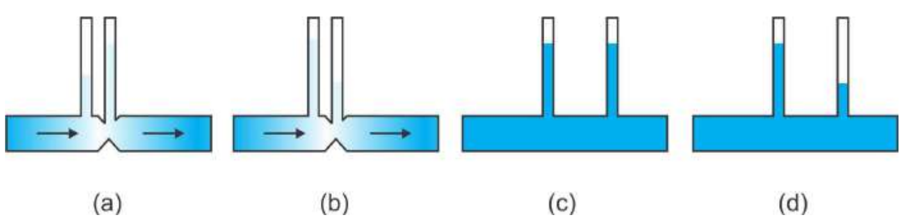


Fig.Q.5.2.1 (a),(b),(c),(d)

Q.5.2.9 The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2. One end of the tube has a number of identical fine holes each of area 1.0 mm^2. The liquid flows inside the tube at 15 cm/s and emerges out of each hole at 60 cm/s. The number of the holes is

(A) 400

(B) 200

(C) 100

(D) 50

Q.5.2.10 A non-viscous, incompressible fluid is in streamlined flow through a narrow pipe of varying cross-section. The pipe is horizontal and its cross section decreases uniformly as shown in Fig.Q.5.2.2. If v_1, v_2 denote the speeds of the fluid at the two ends and P_1, P_2 the respective values of pressure, then

(A) $P_1 > P_2$; $v_1 > v_2$

(B) $P_1 < P_2$; $v_1 < v_2$

(C) $P_1 > P_2$; $v_1 < v_2$

(D) $P_1 = P_2$; $v_1 = v_2$

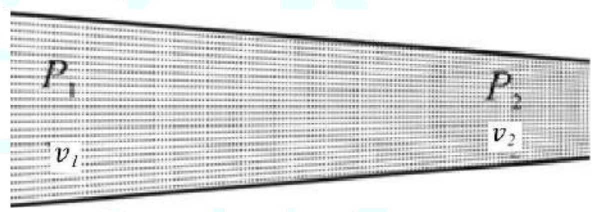


Fig.Q.5.2.2

Module 5.3 Surface Tension

Q.5.3.1. The angle of contact with glass for ethyl alcohol, water, methyl iodide and mercury is 0°, 8°, 30° and 140°. Four identical glass capillaries are immersed in the above liquids. The liquid surface seen in the capillary is convex in case of

(A) ethyl alcohol

(B) water

(C) methyl iodide

(D) mercury

Q.5.3.2 The surface of a liquid stored in a glass container is concave. The angle of contact for the liquid - glass is θ. We have

(A) $F_{\text{Cohesion}} > F_{\text{Adhesion}}$ and $\theta > 90^0$

(B) $F_{\text{Cohesion}} < F_{\text{Adhesion}}$ and $\theta < 90^0$

(C) $F_{\text{Cohesion}} > F_{\text{Adhesion}}$ and $\theta < 90^0$

(D) $F_{\text{Cohesion}} < F_{\text{Adhesion}}$ and $\theta > 90^0$

Q.5.3.3 The surface tension is correctly represented by the relation

(A) Force/area

(B) Energy/area

(C) Force/length

(D) Energy/length

Q.5.3.4 Two mercury droplets of radii 0.1 cm and 0.2 cm coalesce into one single drop. The surface tension of mercury is 435.5×10^{-3} Nm^{-1}. The amount of energy released is

(A) -3.7×10^{-7} J

(B) 3.7×10^{-6} J

(C) 7.4×10^{-6} J

(D) 3.7×10^{-2} J

Q.5.3.5 A square frame of side L is dipped in a liquid of surface tension S. On taking the frame out, a liquid membrane is formed. The force acting on the frame is

(A) $2\,SL$

(B) $4\,SL$

(C) $8\,SL$

(D) $10\,SL$

Q.5.3.6 The work done in increasing the size of a soap film from 10 cm × 6 cm to 10 cm ×11 cm is 2×10^{-4} J. The surface tension of the soap solution is

(A) 2×10^{-2} Nm^{-1}

(B) 2×10^{-6} Nm^{-1}

(C) 2×10^{-4} Nm^{-1}

(D) 4×10^{-2} Nm^{-1}

Q.5.3.7 Two capillary tubes A and B with radii in the ratio 1:2 are immersed in a liquid. A mass M of the liquid rises in tube A. The mass of the liquid that will rise in tube B is

(A) $4M$

(B) $2M$

(C) M

(D) $\frac{M}{2}$

Q.5.3.8 Capillaries of radii 0.2 cm and 0.3 cm are dipped in liquids of surface tensions 40 N/m and 50 N/m respectively. h_1 and h_2 are the heights to which liquids rise in the two capillaries. Taking the angle of contact in the two cases to be the same, h_1/h_2 is

(A) 8/15

(B) 6/5

(C) 5/6

(D) 15/8

Q.5.3.9 The variation of the height h to which a given liquid rises in a capillary of radius r is correctly represented by the graph shown in Fig.Q.5.3.1 (a),(b),(c) and (d)

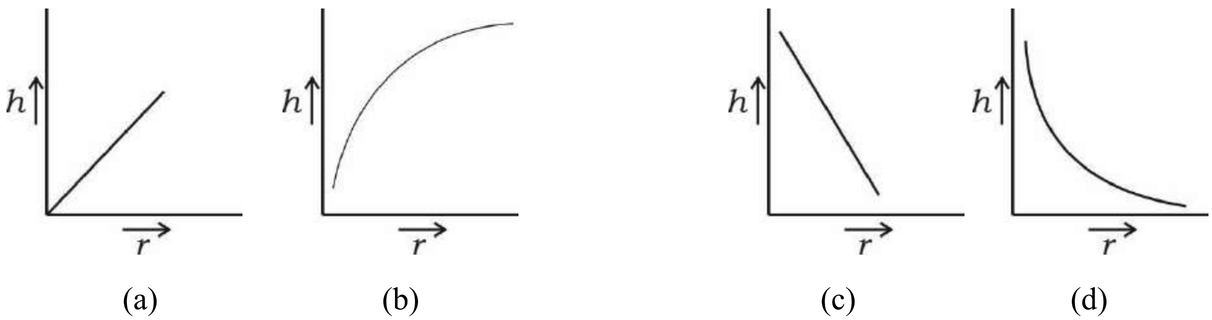


Fig.Q.5.3.1 (a),(b),(c) and (d)

(A) (a)
(B) (b)
(C) (c)
(D) (d)

Q.5.3.10 A liquid of surface tension S rises to a height h_E in a glass capillary tube of radius r on earth. The same liquid in another glass capillary of radius $3r$ rises to a height h_M on the moon. The ratio h_M/h_E is

(A) 2
(B) 1/2
(C) 4
(D) 1/4

ANSWERS

Module 5.1 Pressure

A.5.1.1 (A)

We have $P = hdg$

As g is maximum on the surface of the earth.

Hence the correct option is (A).

A.5.1.2 (C)

We have $P = hdg$

As the density of glycerine is more than that of water, $P_a < P_b$ which is option (C).

A.5.1.3 (C) Total number of marbles on the pan at time $t = n.t$ Mass of the marbles $= nt.m$

Weight of the marbles $= n\ tm.g$

Loss of momentum of the marbles in time $t = (ntm).u$

The pan must gain an equal amount of momentum.

Rate of change of momentum $= n\ mu$

By the second law of motion, the rate of change of momentum is the force.

$\therefore F = n\ mu$

Reading in the scale $= F +$ Weight of the marbles $= nmu + n.\ mtg$

So option (C) is correct.

A.5.1.4 (D)

By definition of pressure, all the units given in (A), (B) and (C) are the same.

So option (D) is correct.

A.5.1.5 (B)

We have, $P_A - P_B = P_B - P_C = P_C - P_D$.

The pressure difference is given by $(\Delta h)\varrho\, g$ provided the density of air remains constant.

We know that the density of air in the atmosphere decreases as height increases.

Hence the pressure difference will be the same in the three cases only if $h_1 < h_2 < h_3$ which is option (B).

A.5.1.6 (C)

Given that $P_A - P_B = P_B - P_C = P_C - P_D.$

The pressure difference is given by $(\Delta h)\, \varrho\, g$

As the density of air is assumed to be constant and the given points are near the earth(so, g is constant), the pressure differences will be equal only if $h_1 = h_2 = h_3$ which is option (C).

A.5.1.7 (A)

The atmospheric pressure at a point is the weight of the air above a surface measured per unit area of the surface. The weight will be the maximum when the point is inside the well.

Hence, option (A) is correct.

A.5.1.8 (D)

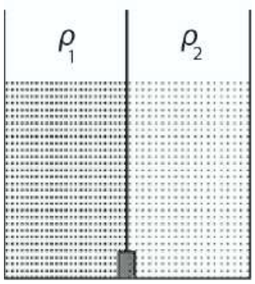
The pressure exerted by a liquid column depends only on the height of the column and not on the shape of the container. $\therefore P_A = P_B.$

But Force = Pressure . area

$\therefore F_B > F_A$ as the vessel B has a larger base area.

Hence, $P_A = P_B; F_A < F_B$ which is option (D).

A.5.1.9 (A) (Fig. A.5.1.1)



Pressure from left side of the door = $h\,\varrho_1\, g$

Pressure from right side of the door = $h\,\varrho_2\, g$

As $\varrho_1 > \varrho_2$; we have $P_{\text{left}} > P_{\text{right}}$.

$\therefore$ Force from the left side exceeds the force from right side of the door.

Hence the door experiences a net force directed towards the right which is option (A). **Fig. A.5.1.1**

A.5.1.10 (B)

In Fig.A.5.1.2 (a), the pressure of the gas in the enclosure exceeds the atmospheric pressure by 20 cm of the liquid column.

In Fig.A.5.1.2 Fig. (b), the pressure of the gas in the enclosure is less than the atmospheric pressure by 18 cm of the liquid column.

$\therefore$ Change in pressure in terms of length of the liquid column is 38 cm which is option (B).

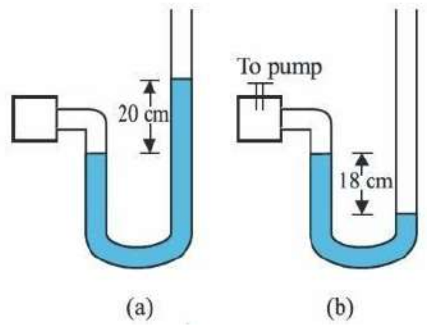


Fig.A.5.1.2

Module 5.2 Bernoulli's Principle

A.5.2.1 (A)

For an ideal fluid, the density remains constant. This rules out options (C) and (D).

A closer packing of streamlines indicates an increase in velocity. Hence, option (A) is correct.

A.5.2.2 (C)

According to the equation of continuity, $A.v$ = constant.

When the radius is doubled, the area of the cross section becomes four times and the speed of flow is reduced to one-fourth. Hence option (C) is correct.

A.5.2.3 (B)

The flow of an ideal fluid tends to be turbulent if the density is low and speed of flow is high.

Hence option (B) is correct.

A.5.2.4 (C)

The mass of liquid crossing any section of liquid in one second is

i.e. (volume/sec). density = constant.

or $Av\varrho$ = constant, v is the speed of flow of the liquid.

As liquid is assumed to be incompressible, so ϱ is constant

$\Rightarrow A.v$ = constant

which is the equation of continuity.

Hence the correct option is (C).

A.5.2.5 (C)

Let v be the speed with which the liquid emerges out of each small hole.

According to the equation of continuity, $A.v = A'v'$.

$R^2.v = N.\ r^2.v'$.

Therefore, $v' = R^2v/Nr^2$

Option (C) is correct.

A.5.2.6 (B) (Fig.A.5.2.1)

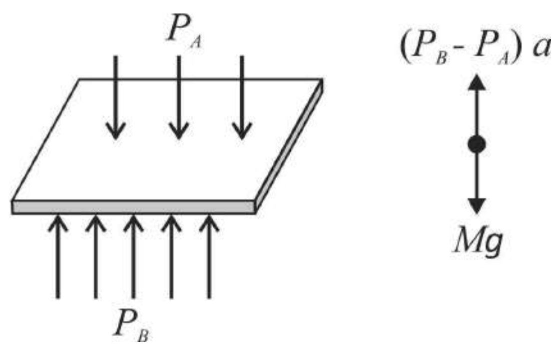


Fig.A.5.2.1

Using Bernoulli's principle, we get $\frac{P_B}{d} + \frac{v^2}{2} = \frac{P_A}{d} + \frac{(nv)^2}{2}$

Therefore, Pressure difference $= P_B - P_A = \frac{(n^2-1)v^2 d}{2}$.

For equilibrium of the plate

$$a(P_B - P_A) = \frac{(n^2-1)av^2 d}{2} = Mg$$

$$\therefore M = \frac{(n^2-1)av^2 d}{2g}$$

Hence Option (B) is correct.

A.5.2.7 (B) According to Bernoulli's principle, the region in which the fluid speed is high, the pressure is low. As the wind speed above the roof is high, the pressure is lower than the pressure below the roof. So, the roof is lifted up and blown away by the storm. Hence option (B) is correct.

A.5.2.8 (D)

In Fig.Q.5.2.1 (b), the speed of the liquid will be high in the narrower part of the tube resulting in reduced pressure.

So the Fig.Q.5.2.1 (b) is correct.

In Fig.Q.5.2.1(c), the area of cross-section is uniform resulting in equal speeds and hence equal pressures at the two points. Hence option (D) is correct.

A.5.2.9 (B)

According to the equation of continuity, we get $A_1v_1 = A_2v_2$.

(no. of holes) $(\frac{1}{100}) \times 60 = 8 \times 15.$

$\therefore$ no. of holes $= \frac{8\times15\times100}{60} = 200.$

Hence option (B) is correct.

A.5.2.10 (C)

According to the equation of continuity, the speed increases as the area decreases.

The speed increases as we move towards the right end of the pipe.

$\therefore v_1 < v_2$

According to Bernoulli's principle, the region in which the fluid speed is high, the pressure is low so $P_1 > P_2$.

Hence option (C) is correct.

Module 5.3 Surface Tension

A.5.3.1 (D)

For the liquid surface to be convex, the angle of contact must be obtuse (>90^0). This is true for mercury in glass. Hence, option (D) is correct.

A.5.3.2 (B)

The liquid surface is concave if the adhesive forces exceed the cohesive forces and the angle of contact is acute.

∴ Option (B) is correct.

A.5.3.3 (B) and (C)

By definition, surface tension = Force/length = Energy/area.

So options (B) and (C) are correct.

A.5.3.4 (B)

Let r_1 and r_2 be the radii of the smaller drops and R the radius of the coalesced drop.

When the drops coalesce, the volume remains constant.

$$\therefore\ (4/3)\pi R^3 = (4/3)\pi r_1^3 + (4/3)\pi r_2^3$$

$$\text{or } R^3 = (0.1)^3 + (0.2)^3 \Rightarrow R = (0.009)^{\frac{1}{3}} \text{ cm.}$$

Decrease in area $= 4\pi r_1^2 + 4\pi r_2^2 - 4\pi R^2 = 4\pi(r_1^2 + r_2^2 - R^2).$

$$= 4\pi(0.0067) = 0.084 \text{ cm}^2$$

Energy released = (surface tension). (decrease in area) = 3.7×10^{-6} J

So, option (B) is correct.

A.5.3.5 (C)

When the frame is taken out, the liquid membrane formed has two free surfaces.

∴ The force on the frame is 8 SL (=2× 4L× S) which is option (C).

A.5.3.6 (A)

Given, work done = 2×10^{-4} J.

The soap film has two free surfaces.

$\therefore$ Increase in area = $2\times[(10\times11) - (10\times6)]$ cm^2 = 100 cm^2 = 10^{-2} m^2

Surface tension = Work/Increase in area = 2×10^{-2} N/m.

Hence, option (A) is correct.

A.5.3.7 (B)

We have $h = \frac{2T\cos\theta}{r\rho g}$.

Given, $r_1/r_2 = 1/2$;

$\therefore h_1/h_2 = 2/1$

Volume of liquid in the capillary $= \pi r^2 h$

$\therefore \frac{V_1}{V_2} = \frac{r_1^2.h_1}{r_2^2.h_2} = \frac{1}{2}$

$\therefore \frac{m_1}{m_2} = \frac{V_1.\rho}{V_2.\rho} = \frac{1}{2} \Rightarrow m_2 = 2m_1 = 2M$

So,option (B) is correct.

A.5.3.8 (B)

We have $h_1 = \frac{2T_1\cos\theta}{r_1\rho g}$

and $h_2 = \frac{2T_2\cos\theta}{r_2\rho g}$.

$\therefore h_1/h_2 = (T_1/T_2).(r_2/r_1) = 6/5.$

Hence option (B) is correct.

A.5.3.9 (C)

We have, $h = \frac{2T\cos\theta}{r\rho g}$.

$\therefore$ The graph of h versus r will be hyperbolic, which is option (C).

A.5.3.10 (A)

On the earth, $h_E = \frac{2\,T\cos\theta}{r\rho g}$.

On the moon, $h_M = \frac{2\,T\cos\theta}{(3r)\rho\frac{g}{6}}$.

$\therefore \frac{h_M}{h_E} = \frac{2}{1}$.

Hence, option (A) is correct.

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6. Thermal Properties of Matter

Module 6.1 Measurement of Temperature and Gas Law

Q.6.1.1 Which among the following statements describes a system in thermal equilibrium?

(A) The total mechanical energy remains constant over a period of time.

(B) A gas confined in a thermally insulated rigid chamber with its pressure, volume, temperature, composition and mass remaining constant over a period of time.

(C) A gas confined in a rigid metal container with its pressure, volume and temperature remaining constant at a given instant of time.

(D) A system where the microscopic parameters of the constituents do not change with time.

Q.6.1.2 The temperature at which Celsius scale and Fahrenheit scale show the same numerical reading is

(A) -30

(B) -40

(C) 30

(D) 40

Q.6.1.3 The temperature of melting ice is

(A) 0 K

(B) -273.16 K

(C) 273.16 K

(D) 373.16 K

Q.6.1.4 A clinical thermometer is kept in mouth for about a minute to ensure that

(A) mercury easily moves across the kink in the thermometer.

(B) the bulb attains thermal equilibrium with the body.

(C) Mercury requires time to expand.

(D) The change in length of the mercury column is proportional to the temperature of the bulb.

Q.6.1.5 A given mass of an ideal gas at temperature 57 °C expands to 150% of its original volume at constant pressure. The final temperature (in degree celsius) is

(A) 495

(B) 358

(C) 220

(D) 222

Q.6.1.6 The standard atmospheric pressure is nearly equal to the pressure exerted by a water column of height 10 m. The volume of an air bubble on the surface of the lake is four times its volume at the bed of the lake. Assuming the temperature to remain the same throughout, the depth of the lake (in meter) is

(A) 20 m

(B) 30 m

(C) 40 m

(D) 50 m

Q.6.1.7 Which of the following statements is correct in the context of the behavior of a substance at 0 K.

(A) The internal energy is the least.

(B) The internal energy is zero.

(C) The particles constituting the substance have no mutual force of attraction and repulsion

(D) Its volume ceases to exist.

Q.6.1.8 The triple point of neon is – 415.44 °F. The corresponding temperatures on Kelvin and Celsius scale are (nearly)

(A) 24. 57 K and – 248.58 °C

(B) 60.12 K and −213.08 °C

(C) 92.12 K and −245.08 °C

(D) 42.35 K and −230.8 °C

Q.6.1.9 The volume of a given mass of a gas at a pressure 150 kPa is 100 cm^3. The pressure on the gas is increased to 300 kPa without change in temperature. The volume of the gas will now be

(A) 200 cm^3

(B) 150 cm^3

(C) 75 cm^3

(D) 50 cm^3

Q.6.1.10 An ideal gas thermometer is governed by the relation,

$T = T_0 + cV^2$,

where T_0 and c are positive constants. V is the volume of one mole of the gas and other symbols have their usual meanings. The value of the minimum attainable pressure is

(A) $R(cT_0)^{1/2}$

(B) $\frac{1}{2}R(cT_0)^{1/2}$

(C) $2R(cT_0)^{1/2}$

(D) $\frac{R}{3}(cT_0)^{1/2}$

Module 6.2 Thermal Expansion

Q.6.2.1 Three rods A, B, and C have the same length, area of cross-section and temperature. The coefficients of linear expansion of the materials of the rods are α_a, α_b and α_c respectively($\alpha_c > \alpha_a > \alpha_b$**).**

The temperature of the three rods is increased by T °C. The change in length L_a, L_b and L_c of the three rods will respectively be

(A) $L_a > L_b > L_c$

(B) $L_a > L_b < L_c$

(C) $L_a < L_b > L_c$

(D) $L_b < L_a < L_c$

Q.6.2.2. The increase in area per unit area of a thin lamina of aluminum for 1 K rise in temperature is

(A) coefficient of linear expansion of the lamina.

(B) coefficient of linear expansion of aluminum.

(C) coefficient of area expansion of aluminum.

(D) coefficient of area expansion of lamina.

Q.6.2.3.The coefficient of linear expansion of brass is 1.8×10^{-5} K^{-1}. The temperature of a meter rod of brass is increased by 50 °C, increase in the length of the rod will be

(A) 0.9×10^{-5} m

(B) 0.09×10^{-5} m

(C) 0.9 mm

(D) 0.09 mm

Q.6.2.4 The temperature of a metal bar of length 100 mm is increased by 100 °C. The length of the bar increases by 0.3 mm. The coefficient of linear expansion of the metal is

(A) 3.0×10^{-5} K^{-1}

(B) 0.3×10^{-5} K^{-1}

(C) 3.0×10^{-3} K^{-1}

(D) 1.8×10^{-5} K^{-1}

Q.6.2.5 The temperature of an aluminum sphere of volume V has increased by 100 K. If the coefficient of volume expansion of aluminum be 7×10^{-5} K^{-1}, the change in volume of the sphere will be

(A) 0.00007 V

(B) 0.0007 V

(C) 0.007 V

(D) 0.07 V

Q.6.2.6 The coefficients of linear expansion of four metals A, B, C and D are 2.5×10^{-5} K^{-1}, 1.8×10^{-5} K^{-1}, 1.7×10^{-5} K^{-1} and 1.2×10^{-5} K^{-1} respectively. The bimetallic strip made from which of these two metals would show maximum bending for the same rise in temperature?

(A) A and B

(B) B and C

(C) C and D

(D) D and A

Q.6.2.7. A metallic tape is calibrated to measure correctly at 20 °C. The coefficient of linear expansion of the metal is 2.9×10^{-5} K^{-1}. The tape measures a distance as 25.000 m on a day when the temperature is 0 °C. The real distance should be

(A) 24.986 m

(B) 24.993 m

(C) 25.000 m

(D) 25. 014 m

Q.6.2.8 The device used for flame test of chemicals in laboratory comprises two materials having same coefficient of linear expansion. The two materials used for this purpose are

(A) Glass and gold.

(B) Gold and silver.

(C) Invar and glass.

(D) Platinum and glass.

Q.6.2.9 The coefficient of linear expansion of brass is $1.8 \times 10^{-5}\ K^{-1}$. The temperature of a given solid brass sphere is increased by 50 °C. The fractional change in the volume is

(A) 0.0009

(B) 0.0027

(C) 0.009

(D) 0.027

Q.6.2.10 Hot air rises up in the atmosphere because the

(A) coefficient of volume expansion of air is quite high.

(B) The density of hot air is less than that of relatively cooler air above it.

(C) The wind blows up the hot air.

(D) Pressure exerted by a gas depends on its volume.

Module 6.3 Applications of Thermal Properties of Matter

Q.6.3.1 Which of the following is correct for pressure P and volume V of an ideal gas at 0 K?

(A) $P \neq 0; V = 0$

(B) $P \neq 0; V \neq 0$

(C) $P = 0; V \neq 0$

(D) $P = 0 ; V = 0$

Q.6.3.2 The heat required to raise the temperature of a body through 1 K

(i) depends upon its mass.

(ii) depends upon its shape.

(iii) depends upon initial temperature.

(iv) depends on the nature of its material.

The incorrect statement/statements out of the above is/are

(A) (i) only.

(B) (ii) only.

(C) (ii) and (iii) only.

(D) All of the above.

Q.6.3.3 Specific heat of a gas at constant pressure is more than its specific heat at constant volume because

(A) a part of the heat supplied is used to keep the volume of the gas constant.

(B) a part of the heat supplied is used for expansion of gas.

(C) Boyle's law does not hold as the temperature is not constant.

(D) The volume of the gas decreases when the pressure is kept constant.

Q.6.3.4 One litre of an oil at 30 °C in a poly-pack floats in water with $1/9^{th}$ of its volume above the water surface. The temperature is decreased by 20 °C. Neglecting any change in volume of the packing material, which of the following statements is correct?

(A) The density of oil decreases.

(B) The packet still floats with $1/9^{th}$ of its volume above the water surface.

(C) The packet floats with more than $1/9^{th}$ of its volume above the water surface.

(D) The packet floats with less than $1/9^{th}$ of its volume above the water surface.

Q.6.3.5 The physical quantity with K^{-1} as its SI unit is

(A) specific heat.

(B) coefficient of volume expansion of a gas.

(C) boiling point of a liquid.

(D) quantity of heat

Q.6.3.6 The correct relation out of the following is

(A) $m = \frac{s\,\Delta T}{\Delta Q}$

(B) $\Delta Q\, m = s\,\Delta T$

(C) $s = \frac{\Delta Q}{\Delta T}$

(D) $s = \frac{\Delta Q}{m\,\Delta T}$

Q.6.3.7 2 kg of water at 70 °C is mixed with 8 kg of water at 30 °C. Assuming no loss of heat to the surroundings, the final temperature of the mixture will be

(A) 63.6 °C

(B) 40 °C

(C) 38 °C

(D) 36 °C

Q.6.3.8 Two liquids A and B have specific heats s_1 and s_2 respectively. 40 g of A at 10 °C is mixed with 25 g of B at 60 °C. The equilibrium temperature of mixture is 30 °C. The ratio of thermal capacities of A and B is

(A) 8:5

(B) 15 : 16

(C) 2:3

(D) 3:2

Q.6.3.9 C_P and C_V are the molar specific heats of a gas. Given y (>0) is the difference and z (>1) is the ratio of the two specific heats of the gas. Which of the following is correct?

(A) $C_V = \frac{y}{(z-1)}$, $C_P = \frac{yz}{(z-1)}$

(B) $C_P = \frac{y}{(z-1)}$, $C_V = \frac{y}{(z+1)}$

(C) $C_V = \frac{yz}{(z-1)}$, $C_P = \frac{y}{(z+1)}$

(D) $C_P = \frac{yz}{(z-1)}$, $C_V = \frac{yz}{(z-1)}$

Q.6.3.10. Which material is most suitable for making handles of cooking vessels?

(A) Low heat capacity and high conductivity.

(B) High heat capacity and high conductivity.

(C) Low heat capacity and low conductivity.

(D) High heat capacity and low conductivity.

ANSWERS

Module 6.1 Measurement of Temperature and Gas Laws

A.6.1.1 (B)

Option (B) is the statement that describes the conditions which a system in thermal equilibrium should possess. Option (A) refers to the total energy; option (C) does not include composition while option (D) refers to microscopic parameters.

A.6.1.2 (B)

Let the temperature be x

Using the conversion formula $\frac{F-32}{180} = \frac{C}{100}$, we have

$\frac{x-32}{180} = \frac{x}{100}$

Solving, we get $x = -40$

A.6.1.3 (C)

Option (A) is incorrect as the melting point of ice is 0 °C and not 0 K.

Option (B) is incorrect as no temperature is possible below 0 K.

Option (D) refers to boiling point of water.

A.6.1.4 (B)

Mercury in the bulb requires some time to attain thermal equilibrium with the body.

A.6.1.5 (D) According to Charles' law $\frac{V_1}{T_1} = \frac{V_2}{T_2}$. Therefore,

$T_2 = T_1\,(V_2/V_1)$,

or $T_2 = (273 + 57)\,(1.5) = 495$ K

495 K = (495-273) °C = 222 °C

A.6.1.6 (B)

$P_{\text{bed}} V_{\text{bed}} = P_{\text{surface}} V_{\text{surface}}$.

Therefore , $P_{\text{bed}} = V$ = (1atm) (4 V).

Therefore, P_{bed} = 4 atm = 40 m of water column.

Depth of the lake = 40 m − 10 m = 30 m.

A.6.1.7 (A)

From the kinetic theory of gasses, we know that the internal energy of an ideal gas is zero at 0 K.

But for every substance there is a zero point energy, and its value is non-zero and the least at 0 K.

So, (A) is correct but (B) and (C) are incorrect.

(D) holds for an ideal gas. It is not generally true for any substance.

A.6.1.8 (A)

Use the formula $\frac{F-32}{180} = \frac{K-273.15}{100} = \frac{C}{100}$, we get

Temperature in °C= −248.58

Temperature in K= 24.57

A.6.1.9 (D)

At constant temperature, $P_1 V_1 = P_2 V_2$

or, $V_2 = P_1 V_1 / P_2$

Therefore V_2 = Volume at 300 kPa = (150 × 100)/300 = 50 cm^3

A.6.1.10 (C)

$$P = \frac{RT}{V} = \frac{R}{V}(T_0 + cV^2) = \frac{RT_0}{V} + RcV$$

so $P = x + y$ where $x = \frac{RT_0}{V}$ and $y = RcV$

so, $xy = RcT_0 = k$ = constant

Now $(x + y)^2 = (x - y)^2 + 4xy = (x - y)^2 + 4k$

(x+y) is minimum when the perfect square $(x - y)^2 = 0$ or $x = y$

$$\frac{RT_0}{V} = RcV$$

$$V^2 = \frac{T_0}{c}$$

$$V = (\frac{T_0}{c})^{1/2}$$

so $P_{min} = \frac{2RT_0}{V} = 2RT_0(\frac{c}{T_0})^{1/2} = 2R(cT_0)^{1/2}$

which is option(C) .

Module 6.2 Thermal Expansion

A.6.2.1 (D)

Increase in length of the three rods varies directly as their coefficients of thermal expansion. Initial dimensions and the change in temperature are the same.

A.6.2.2 (C)

Option (C) defines the coefficient of area expansion of the material of the lamina, that is, aluminum. Options (A) and (B) refer to coefficient of linear expansion while option (D) refers to coefficient of expansion of lamina and is therefore incorrect.

A.6.2.3 (C)

The change in length of the brass rod ΔL will be equal to 1000 mm × 50 °C × 1.8 ×10^{-5} K^{-1} = 0.9 mm.

A.6.2.4 (A)

Use the relation, $\alpha_l = \frac{1}{\Delta T}\frac{\Delta l}{l}$

A.6.2.5 (C)

Use the relation, $\alpha_V = \left(\frac{\Delta V}{V}\right)\frac{1}{\Delta T}$

A.6.2.6 (D)

The extent of bending of a bimetallic strip depends on the difference in the coefficients of linear expansion of its constituent metals. Therefore, option (D) is correct.

A.6.2.7 (A)

The tape has been used at 0 °C while it has been calibrated for 20 °C. So, the markings on the tape are closer together. As a result, this measurement is more than the actual length. The error is 2.9 × 10^{-5} K^{-1}×25 m × (−20

K) = − 0.0145 m (rounded to 0.014). Hence the real distance = (25 – 0.014) = 24.986 m. So Option (A) is correct.

A.6.2.8 (D)

This question tests the knowledge that coefficients of linear expansions of glass and platinum have the same value. For the two materials in Options (A), (B) and (C), the coefficients of expansion are different .

A.6.2.9 (B)

Use $\Delta V/V = \alpha_V \times \Delta T = 3\,\alpha_L \times \Delta T$

A.6.2.10 (B)

The volume of a gas increases when its temperature is increased. Therefore, the density of a gas decreases with the rise in its temperature.

Module 6.3 Applications of Thermal Properties of Matter

A.6.3.1 (D)

At 0 kelvin both the pressure and volume of an ideal gas are zero.

A.6.3.2 (B)

We have $\Delta Q = m.\ C.\ \Delta T$. Therefore, ΔQ depends on both mass and specific heat but not on the shape. However, ΔQ, depends on the initial temperature as the specific heat of a material varies with temperature.

A.6.3.3 (B)

When a gas is heated at constant pressure, a part of the energy supplied is used in doing work for expansion of the gas.

A.6.3.4 (D)

The volume of oil decreases on cooling and the density increases. Hence the fraction of the volume of the oil packet above the water surface decreases.

A.6.3.5 (B)

The SI unit of coefficient of volume expansion is K^{-1}. The SI unit of specific heat and the quantity of heat are J $kg^{-1}\ {}^{\circ}C^{-1}$ and J respectively while the unit of boiling point is K or °C.

A.6.3.6 (D)

The specific heat is defined as the quantity of heat required to raise the temperature of unit mass of a substance by unit temperature, that is, $s = \frac{\Delta Q}{m\Delta T}$. Other three Options are permutations of the same relation.

A.6.3.7 (B)

The heat gained by 2 kg water at 30 °C to attain the temperature T of the mixture will be equal to the heat lost by water at 70 °C in attaining the temperature T.

The temperature T of the mixture of water is given by $T = (m_1sT_1 + m_2sT_2)/\ (m_1s + m_2s)$.

Substituting m_1 = 2 kg, m_2 = 8 kg, T_1 = 70 °C and T_2 = 30 °C, we. get T = 38 °C.

A.6.3.8 (D)

Let C_1 and C_2 be the thermal capacities of liquid A and B. From principle of calorimetry

$C_1(30 - 10) = C_2(60 - 30)$

$$\therefore \frac{C_1}{C_2} = \frac{3}{2}$$

A.6.3.9 (A)

We know that $C_P > C_V$. The difference y is a positive number. Therefore,

$$C_P - C_V = y \tag{6.3.1}$$

The ratio, z of the two is more than one. Therefore

$$\frac{C_P}{C_V} = z \tag{6.3.2}$$

From Eqs(6.3.1) and (6.3.2), we have

$$C_V = \frac{y}{(z-1)}$$

and $C_P = \frac{yz}{(z-1)}$

A.6.3.10 (D)

Heat capacity is defined as the amount of heat required to raise the temperature of a substance by one unit. Since the handle of a cooking vessel should not get heated easily, it should have a high heat capacity.

The handle of a cooking vessel should be a thermal insulator, hence it should have low conductivity.

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7. Thermodynamics

Module 7.1 Scope of Thermodynamics

Q.7.1.1 Two small objects A and B (with A smaller of the two) are kept in contact, with a large object C such that they are not in direct contact with each other for a sufficient time. If A and B are now kept in contact with each other, there would be

(A) a flow of heat from A to B.

(B) a flow of heat from B to A.

(C) no flow of heat between A and B as their total heat contents would have identical values.

(D) no flow of heat between A and B as their temperatures would have identical values.

Q.7.1.2 The sequence of development of the three laws (zeroth, first and second law) is

(A) first, second, zeroth

(B) zeroth, first, second

(C) second, first, zeroth

(D) second, zeroth, first

Q.7.1.3 A thermodynamic variable is said to be a state variable if its value depends only on the state of the system and not on the way that state has been arrived at.

Out of the following:

(Heat, work, internal energy)

the state variable/ variables, is/are

(A) Heat and work

(B) Work and internal energy

(C) Internal energy

(D) Heat

Q.7.1.4 The temperature of a system may be defined as a

(a) characteristic property of the system that determines whether or not this system would be in thermal equilibrium with another system.

(b) measure/an indicator of the kinetic energy of its molecules.

We refer to definitions (a) and (b),

(A) as, the kinetic theory based and the thermodynamic definitions of temperature, respectively.

(B) as the thermodynamic, and the kinetic theory based definitions of temperature, respectively.

(C) both as the kinetic theory-based definitions of temperature.

(D) both as the thermodynamic definitions of temperature.

Q.7.1.5 Two bodies A and B are in thermal equilibrium with each other. The volume of body A is four times that of body B. Assume that the heat contents of the two bodies are proportional to their respective volumes. The net amount of heat transferred, from A to B, when the two bodies are put in thermal contact with each other, would be

(A) 3/2 times the heat content of B
(B) 3 times the heat content of B
(C) 3/8 times the heat content of A
(D) Zero

Q.7.1.6 A person, of mass M kg, wants to lose m kg ($m \ll M$) by going up and down on stairs of height H. Assume that he burns twice as much fat while going up than coming down and 1 kg of fat gets burnt on expending Q kilocalorie (1 calorie ≈ 4.2 joule). The number of cycles (of going up and down) that he needs to undertake, to achieve his aim, would then equal:

(A) $\frac{2800mQ}{MgH}$

(B) $\frac{4200\, mQ}{MgH}$

(C) $\frac{667\, mQ}{MgH}$

(D) $\frac{1000\, mQ}{MgH}$

Q.7.1.7 The internal energy, of any general system,

(A) can change only when the temperature of the system (ideal or practical) changes.

(B) can change only when the potential energy of the system (ideal/practical) changes.

(C) can change either when the temperature or the potential energy of the system (ideal/practical) changes.

(D) never changes irrespective of the changes it may undergo.

Q.7.1.8 The thermodynamic state of a system is described in terms of the values of its (usual), three state variables – the pressure (*P*), volume (*V*), and temperature (*T*).

If one of the thermodynamic states of a given system is written as (P_0, V_0, T_0), the system would be said to be in a new thermodynamic state.

(A) If, and only if, its state variables in this (new) state, have values nP_0, nV_0, nT_0 (n = an integer).

(B) If and only if, all its three state variables in this (new) state, have values different from their initial values.

(C) If and only if, at least two of its state variables in this (new) state, have values different from their initial value.

(D) If any one of its state variables, in this (new) state, has a value different from its initial value.

Q.7.1.9 System A is a hot light metal bar; system B is a large bucket of warm water and system C is the surrounding air. On putting them in mutual contact, heat would flow from system

- **(A)** B to system A
- **(B)** C to system B
- **(C)** C to system A
- **(D)** System A to system B, as well as from system A to system C

Q.7.1.10 Internal energy of a system remains constant, when

- **(A)** its pressure remains constant.
- **(B)** its heat content remains constant.
- **(C)** its volume remains constant.
- **(D)** its temperature remains constant.

Module 7.2 Thermodynamic Processes

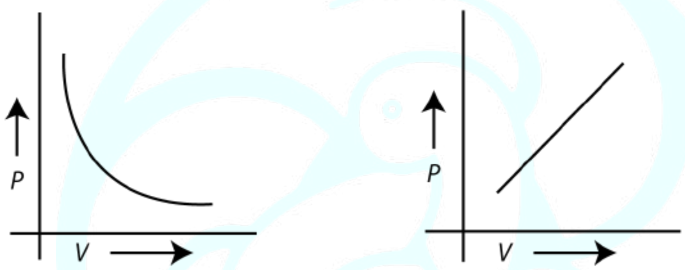
Q.7.2.1 The pressure, volume and temperature of an ideal gas have values (P_0, V_0, T_0) in its initial state. It undergoes an expansion so that its volume becomes V ($V > V_0$), in two different ways. The final pressure, volume and temperature values, in the two cases, are

(i)(P, V, T_0)

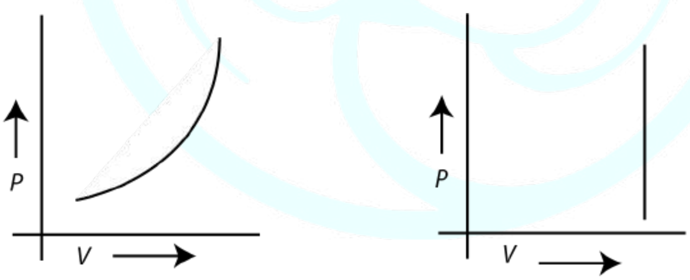
(ii)(P_0, V, T)

The (P-V) diagrams, for the two cases (Fig.Q.7.2.1), then correspond, respectively, to

(A)



(B)



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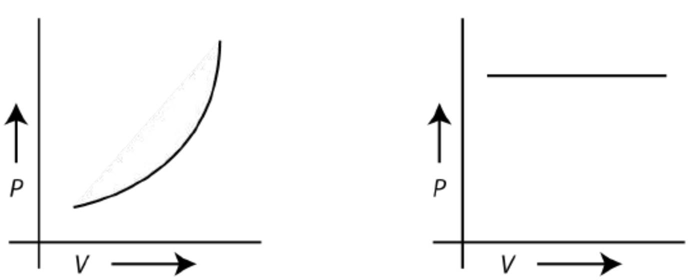

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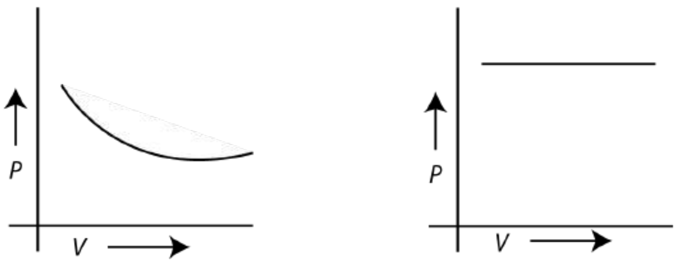


Fig.Q.7.2.1

Q.7.2.2 Heat is added to a system but its temperature does not change. This could be so because the system must have undergone an

- **(A)** isothermal change
- **(B)** isobaric change
- **(C)** isochoric change
- **(D)** adiabatic change

Q.7.2.3 Consider an ideal gas expanding through a process in which the pressure (*P*) and volume (*V*) are related as per the equation.

$$P = aV^3 + bV^2 + cV + d$$

The ratio of the slope of the *P* - *V* graph for this process, to that for an adiabatic process, at some point (V_0, P_0), would be

- **(A)** $-\frac{1}{\gamma p_0}\left[3aV_0^{\ 2} + 2bV_0 + c\right]$
- **(B)** $-\left[\frac{3aV_0^{\ 3}+2bV_0^2+cV_0}{\gamma P_0}\right]$
- **(C)** $\gamma P_0\left[3aV_0 + 2b + cV_0^{\ -1}\right]$

(D) $\gamma P_0\left[3aV_0^{\ 2} + 2bV_0 + c\right]$

Q.7.2.4 The walls of a cylinder are made of a heat insulator and the movable piston, present in it, is insulated by keeping a light pile of sand on it.

The cylinder initially contains 3 moles of hydrogen at standard temperature and pressure. The piston is moved to compress the gas to half its original volume. The factor, by which the pressure of the gas would increase, is

(A) $2^{7/5}$

(B) $2^{5/3}$

(C) $2^{5/5}$

(D) $2^{7/3}$

Q.7.2.5 Two identical containers A and B each of volume V, contain, respectively,

(i) a monatomic (ii) a diatomic gas, at the same pressure and temperature.

The gas in container A is compressed to a volume ($V/4$) isothermally while that in container B is compressed to a volume $V/8$ adiabatically. The ratio of the final pressure in container B, to that in container A, would be

(A) $(2^{11/5})$

(B) 16

(C) 8

(D) $(2^{16/5})$

Q.7.2.6 The tube, in a cycle tyre of (fixed) volume V, is being filled with air from a pump. At each stroke of the pump, a (small) volume ΔV air is transferred to the tube adiabatically. Assuming air to be a diatomic gas, the work done, in increasing the air pressure in the tube from P_1 to P_2, is (nearly)

(A) $5/7\ V(P_2 - P_1)$

(B) 7/5 $V(P_2 - P_1)$

(C) $\frac{5}{3}V(P_2 - P_1)$

(D) $\frac{3}{5}V(P_2 - P_1)$

Q.7.2.7 Imagine a heat engine following a cycle, represented by the *P-V* diagram shown in Fig.Q.7.2.2.

This cycle is made up of (i) an isothermal from state X to state Y (ii) an adiabatic from state Y to state Z and (iii) an adiabatic from state Z to state X.

Such a cycle is not permissible because

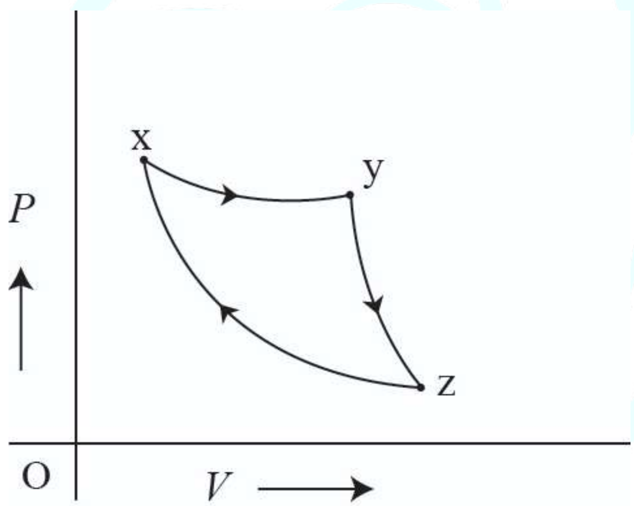


Fig.Q.7.2.2

(A) It shows an isothermal (XY) intersecting with an adiabatic (YZ).

(B) It represents a cycle in which mechanical energy gets completely converted into heat which is not in accord with the second law of thermodynamics.

(C) It represents a cycle in which heat gets completely converted into mechanical energy which is not in accord with the second law of thermodynamics.

(D) It represents a cycle in which the internal energy (U) of the system in all the three states have different values, i.e., $U_X \neq U_Y \neq U_Z$.

Q.7.2.8 One mole of a perfect diatomic gas is taken adiabatically from an initial state (P, V, T) to a final state (P, 32 V, T'). The ratio $\left(\frac{T'}{T}\right)$ equals

(A) $2^{\frac{5}{3}}$

(B) $2^{\frac{25}{3}}$

(C) 2^{-2}

(D) 2^{-4}

Q.7.2.9 The *PV* diagram, of the cycle followed by a heat engine, having one mole of a perfect gas (in its cylinder - piston set up), is as shown in Fig. Q.7.2.3. The net work done by the engine, in one cycle is

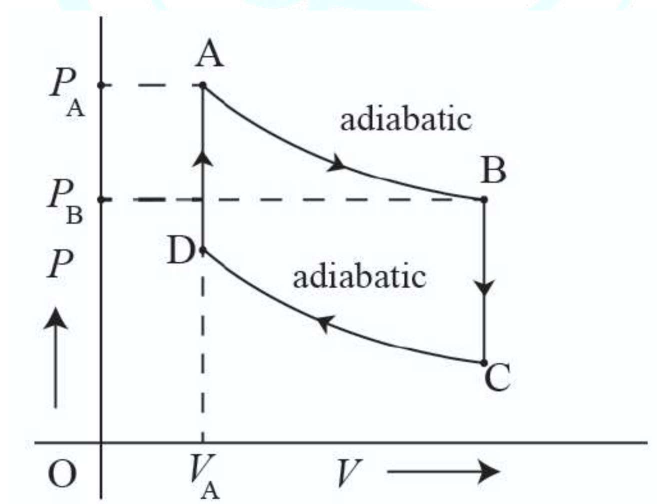


Fig.Q.7.2.3

(A) $\frac{1}{(1-\gamma)}\left[V_A\left(P_D - P_A\right) + V_B\left(P_B - P_C\right)\right]$

(B) $\frac{1}{(1-\gamma)}\left[V_A(P_B - P_A) + V_B(P_C - P_D)\right]$

(C) $\frac{1}{(1-\gamma)}\left[V_A(P_B + P_A) - V_B(P_C + P_D)\right]$

(D) $\frac{1}{(1-\gamma)}\left[V_A(P_D + P_A) - V_B(P_B + P_C)\right]$

Q.7.2.10 The *P*-*V* diagram, for a process of an ideal gas, has the form shown here (Fig.Q.7.2.4).

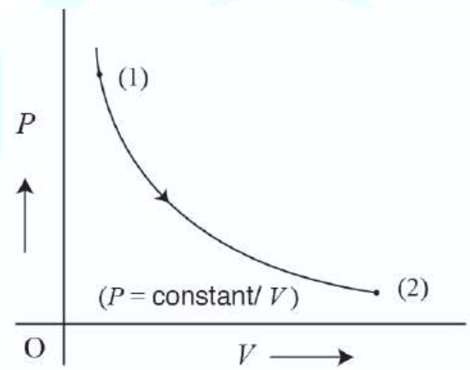
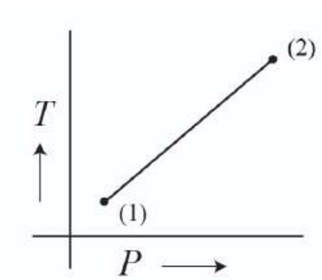


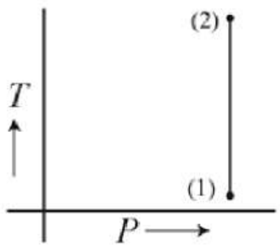
Fig.Q.7.2.4

The (*T*-*P*) diagram for this gas would have the form

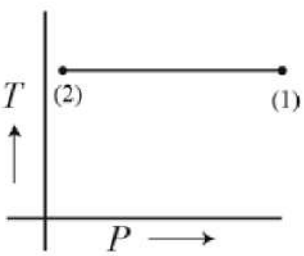
(A)



(B)



(C)



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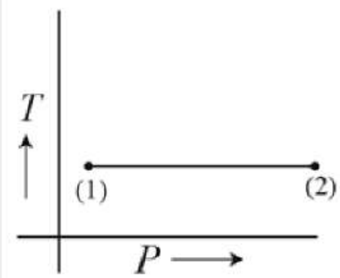


Q.7.2.11 The *P*-*V* diagram, for an ideal gas, has the form shown in Fig.Q.7.2.5.

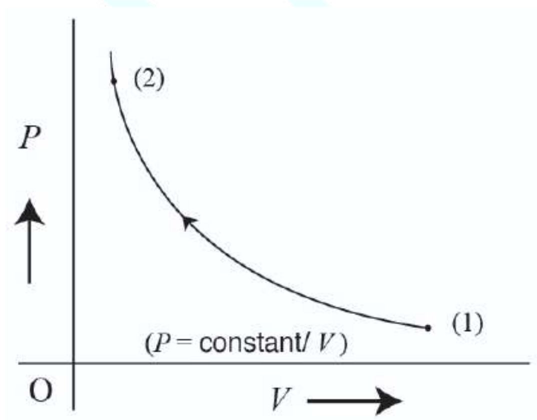
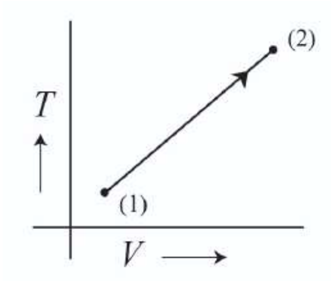
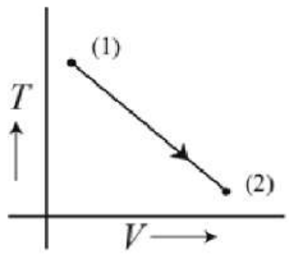
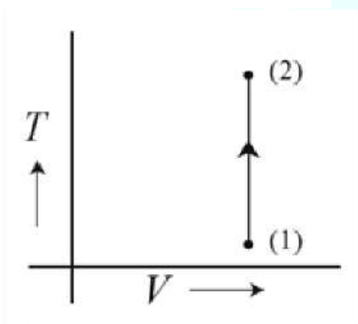


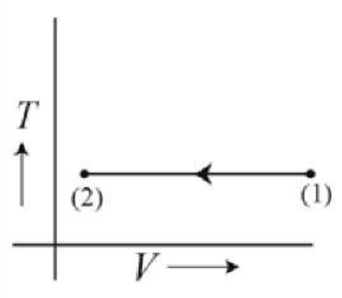
Fig.Q.7.2.5

The (*T*-*V*) diagram. For this gas, would have the form

(A) 

(B) 

(C) 

(D) 

Module 7.3 First Law of Thermodynamics

Q.7.3.1 A system goes from a state X to another state Y by two different Paths, as shown in its $P-V$ diagram (Fig.Q.12.3.1). The heat given to the system, along Path 1 is 1000 J while the work done by the system, along Path 1, exceeds the work done by it along Path 2, by 100 J. The heat given to the system, along Path 2,

(A) cannot be calculated from the given data

(B) equals 1100 J

(C) equals 900 J

(D) would still be 1000 J as heat exchanged, between two given states, is a Path independent variable.

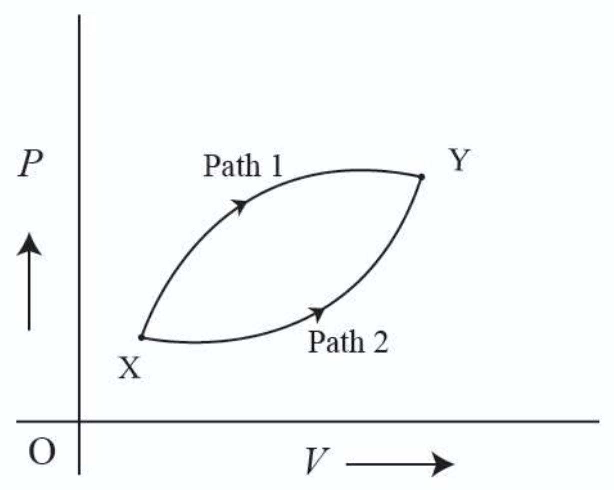


Fig.Q.7.3.1

Q.7.3.2 In the usual statement of the first law of thermodynamics, namely,

$$dQ = dU + dW$$

(A) dU is (+ve) when the temperature of the system decreases and dW = (− ve) when work is done by the system.

(B) dU is (− ve) when the temperature of the system decreases and dW = (− ve) when work is done by the system.

(C) dU is (+ve) when the temperature of the system increases and dW = (+ve) when work is done by the system.

(D) dU is (− ve) when the temperature of the system increases and dW = (− ve) when work is done by the system.

Q.7.3.4 An electric heater supplies heat to a system at a rate of 100 joule per second. If the system

(i) does work at the rate of 75 joule per second

(ii) has work done on it at the rate of 75 joule per second,

the internal energy of the system

(A) would increase at the rate of 25 joule per second in both the cases.

(B) would increase at the rate of 25 joule per second in case (i) and at the rate of 175 joule per second in case (ii).

(C) would increase at the rate of 175 joule per second in case (i) and at the rate of 25 joule per second in case (ii).

(D) would decrease at the rate of 25 joule per second in case (i) and would increase at the rate of 25 joule per second in case (ii).

Q.7.3.5 A system goes from an equilibrium state A to another equilibrium state B via an adiabatic process. During this process, work, equal to 22.3 J, is done on the system.

If this system were to be taken from the state A to state B, via another process in which the net heat absorbed by the system is 39.2 J, the net work done by the system would be

(A) 16.9 J

(B) 62.3 J

(C) 22.3 J

(D) 39.2 J

Q.7.3.6. The specific heat capacity of a given gas, at constant pressure, equals (5/2) $R.\eta$ Moles of this gas are kept in a closed vessel of fixed volume V.

A possible name of this gas, and the rise in its temperature when an amount of heat Q is added to it, (at constant volume) are, respectively,

(A) Hydrogen; $\left(\frac{2}{5}\ \frac{Q}{\eta R}\right)$

(B) Oxygen; $\left(\frac{2}{7}\ \frac{Q}{\eta R}\right)$

(C) Neon; $\left(\frac{2}{3}\ \frac{Q}{\eta R}\right)$

(D) Helium; $\left(\frac{2}{7}\ \frac{Q}{\eta R}\right)$

Q.7.3.7. When an amount of heat ΔQ is given to a system, the system goes from an initial state A to a final state B. During this transition, the system does a work ΔW and its internal energy changes by an amount ΔU. This transition of the system can be brought about in a variety of ways. We can, however, say that in this transition.

(A) ΔQ remains the same irrespective of the way the transition is brought about; ΔU and ΔW, however, depend on the path followed.

(B) ΔW remains the same irrespective of the way the transition is brought about; ΔU and ΔQ, however, depend on the path followed.

(C) ΔU remains the same irrespective of the way the transition is brought about; ΔQ and ΔW, however, depend on the path followed.

(D) ΔQ, ΔW, ΔU all depend on the way the transition is brought about.

Q.7.3.8. One mole of a monatomic (ideal) gas is taken from a point K to another point L, as shown on its *P-V* diagram (Fig.Q.7.3.2). The temperature of the gas, at point K, is T_0. The change in the internal energy of the gas, during this transition, would equal

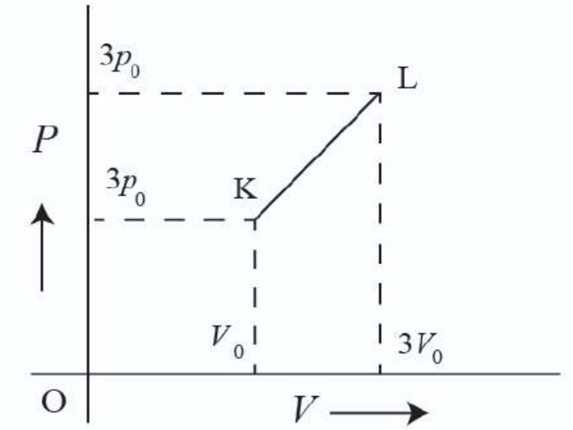


Fig.Q.7.3.2

(A) 20 RT_o

(B) 16 RT_o

(C) 12 RT_o

(D) 8 RT_o

Q.7.3.9 An ideal gas undergoes a change of state from a state A to another state B in four different ways - as per the *P-V* diagram shown here(Fig.Q.7.3.3). If the changes in the internal energy of the gas, in the four cases, are denoted by ΔU_1, ΔU_2, ΔU_3, ΔU_4, we would have

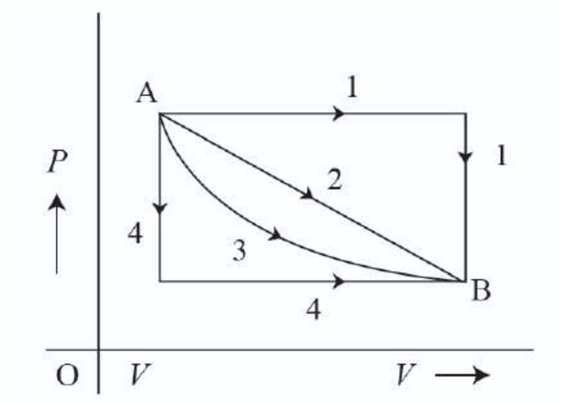


Fig.Q.7.3.3

(A) $\Delta U_1 > \Delta U_2 > \Delta U_3 > \Delta U_4$

(B) $\Delta U_1 > \Delta U_2 = \Delta U_3 > \Delta U_4$

(C) $\Delta U_1 < \Delta U_2 = \Delta U_3 < \Delta U_4$

(D) $\Delta U_1 = \Delta U_2 = \Delta U_3 = \Delta U_4$

Q.7.3.10 An ideal gas undergoes a change of state from a state A to another state B in three different ways (indicated by x, y, and z in Fig.Q.7.3.4 shown below). If the works done by the gas, in the three cases, are denoted by W_x, W_y and W_z we would have

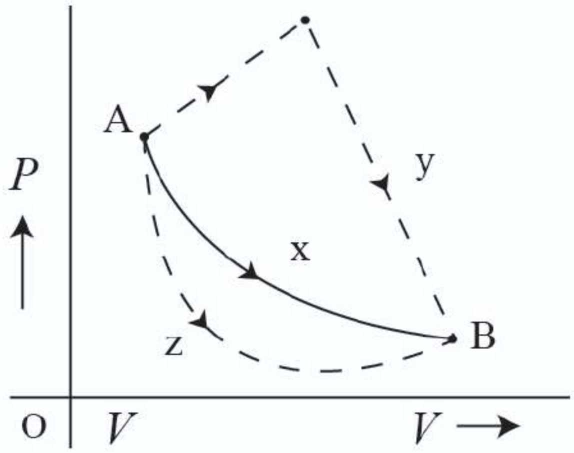


Fig.Q.7.3.4

(A) $W_x = W_y = W_z$

(B) $W_x > W_y > W_z$

(C) $W_y > W_x > W_z$

(D) $W_y > W_z > W_x$

Q.7.3.11 The temperature of the gas, present in a container, was increased without adding any heat to it. This change was brought about through an

(A) isobaric expansion

(B) isochoric compression

(C) adiabatic compression

(D) adiabatic expansion

Module 7.4 Second Law of Thermodynamics

Q.7.4.1 The coefficient of performance of a refrigerator is 9 and it is working in a room where the surrounding temperature is 27 ^{0}C. The temperature, inside this refrigerator, is

(A) 0 ºC
(B) −1 ºC
(C) −2 ºC
(D) −3 ºC

Q.7.4.2 The coefficient of performance of a practical refrigerator, working between an inside temperature of -3ºC and a surrounding temperature of 27 ºC, is 50% of that of an ideal refrigerator working between the same two temperatures. The electric motor, providing the needed mechanical work, has a power of 1 kW.

The heat, taken out of this refrigerator, per second, equals

(A) 9.0 kJ
(B) 4.5 kJ
(C) 18 kJ
(D) 13.5 kJ

Q.7.4.3 The cycle followed by an engine (having one mole of a perfect gas in its cylinder-piston set up) is represented by the *P*- *V* diagram shown in Fig.Q.7.4.1. Given that $C_v = \frac{3}{2}R$, the heats exchanged by the engine with the surroundings, for each section of its cycle (in the order AB, BC, CD, DA) are equal, respectively, to

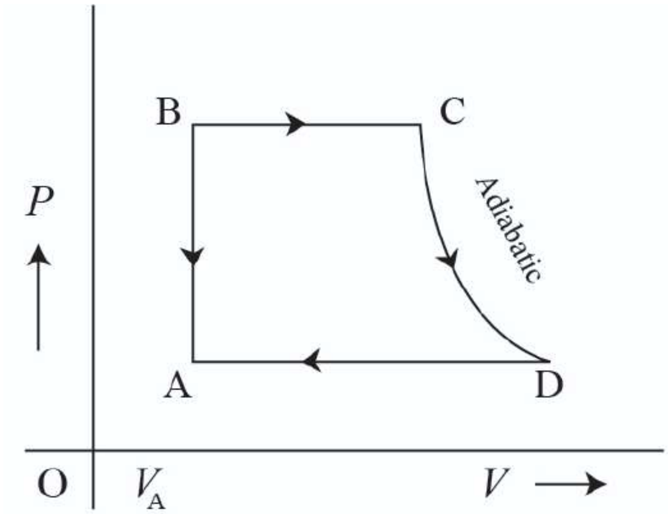


Fig.Q.7.4.1

(A) $\left[\frac{3}{2}V_A\left(P_B - P_A\right)\right], \left[\frac{5}{2}P_B\left(V_C - V_B\right)\right]$, 0 and $\frac{5}{2}\left[P_A\left(V_A - V_D\right)\right]$

(B) $\left[\frac{5}{2}V_A\left(P_B - P_A\right)\right], \left[\frac{3}{2}P_B\left(V_c - V_B\right)\right]$, 0 and $\left[\frac{5}{2}P_A\left(V_A - V_D\right)\right]$

(C) $\left[\frac{3}{2}V_A\left(P_B - P_A\right)\right], \left[\frac{5}{2}P_B\left(V_C - V_D\right)\right]$, 0 and $\frac{3}{2}\left[P_A\left(V_A - V_D\right)\right]$

(D) $\left[\frac{3}{2}V_A\left(P_B - P_A\right)\right], \left[\frac{3}{2}P_B\left(V_C - V_D\right)\right]$,0 and $\left[\frac{3}{2}P_A\left(V_A - V_D\right)\right]$

Q.7.4.4. The surrounding temperature, for a refrigerator, equal 36°C. The temperature, is maintained at 9°C. The coefficient of performance is

(A) 11.4

(B) 10.4

(C) 4.0

(D) 1.9

Q.7.4.5. The coefficient of performance of a given refrigerator, operating in a room having a temperature of 27 ºC, equal 5. The temperature, inside the refrigerator, is

(A) – 33ºC

(B) – 23ºC

(C) – 13ºC

(D) – 3ºC

Q.7.4.6. A Carnot engine is working between a heat source of a temperature T_1 = 500 K and a heat sink maintained at a temperature T_2 = 300 K. The mechanical work produced by the engine, per cycle, is 1 kJ. The heat, H_1, taken in by the engine from the source and the heat, H_2, given by the engine to the sink are then equal, respectively, to

(A) 5.5 kJ and 4.0 kJ

(B) 3.5 kJ and 2.5 kJ

(C) 2.5 kJ and 1.5 kJ

(D) 2.0 kJ and 1.0 kJ

Q.7.4.7. A heat engine, operating between a 'sink', at a temperature of 27 ºC, and a 'source' at a temperature of (327 ºC), produces 1.4 kJ of mechanical work per cycle per calorie (1 calorie = 4.2 J) of heat absorbed from the source. The ratio of the efficiency of this engine, to that of an ideal heat engine, operating between the same 'source' and 'sink' temperatures, is

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

Q.7.4.8. Consider a heat engine as shown in Fig.Q.7.4.2. Q_1 and Q_2 are heat is added to a heat bath at temperature $T_1$and heat taken from another bath at temperature, T_2 during one cycle of an engine. W is the mechanical work done on the engine.

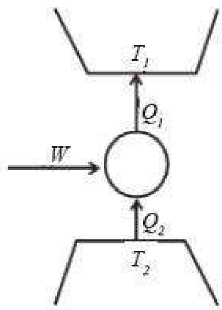


Fig.Q.7.4.2

If $W > 0$, the possibilities are:

(A) $Q_1 > Q_2 > 0$

(B) $Q_2 > Q_1 > 0$

(C) $Q_2 = Q_1 < 0$

(D) $Q_1<0, Q_2>0$

Q.7.4.9 The inside of an ideal refrigerator is at a temperature t_1 ºC, its surroundings are at a temperature t_2 ºC. the coefficient of performance of this refrigerator is (very nearly) equal to one; if

(A) $t_1 = 2t_2$

(B) $t_1 = 2t_2 + 273$

(C) $t_1 = t_2 + 273$

(D) $t_1 = 2(t_2 + 273)$

Q.7.4.10 The heat source and the heat sink, being used with an ideal heat engine, are maintained at temperatures of t_1 ºC and t_2 ºC, respectively. The efficiency of this (ideal) heat engine would (very nearly) equal 25%, if

(A) $t_1 = 4t_2$

(B) $t_1 = \frac{4}{3}t_2 + 273$

(C) $t_1 = \frac{4}{3}t_2 + 91$

(D) $t_1 = \frac{4}{3}t_2 + 364$

ANSWERS

Module 7.1 Scope of Thermodynamics

A.7.1.1 (D)

As per the zeroth law of thermodynamics, if two objects are individually in thermal equilibrium with a third object C, they are also in thermal equilibrium with each other. This implies that their temperatures acquire identical values (and there is no flow of heat between them when they are kept in contact with each other).

A.7.1.2 (A)

The zeroth law of thermodynamics was developed after the development of the first and second laws (in that order). It was for this reason that it was named as the zeroth law (as it was regarded as something needed before the first and second laws) and not the third or fourth law of thermodynamics.

A.7.1.3 (C)

The heat exchanged/work done, in going from a state 1 to another state 2, depends on the way/path followed in bringing about this change. The internal energy change, however, is independent of the path followed. Therefore, out of the given three thermodynamic variables, only internal energy is a state variable.

A.7.1.4 (B)

Definition (a), based on the zeroth law of thermodynamics, is the thermodynamic definition of temperature.

Definition (b), is based on the kinetic theory of gasses.

A.7.1.5 (D)

Since the bodies are in thermal equilibrium, they are at the same temperature. Hence there would be no heat transfer between them when they are put in contact with each other.

A.7.1.6 (D)

Let the energy spent in going up be q joule; the energy spent in going down is then (q/2) joule.

Now $q = Mgh$

∴ Total energy spent in one cycle is 3/2 MgH.
The energy that he needs to spend (in achieving his aim) is
$m(Q \times 10^3 \times 4.2)$

Hence the number of cycles that he needs to undertake is $\frac{m(Q\times10^3\times4.2)}{(3/2\ MgH)} = \frac{2800mQ}{MgH}$

A.7.1.7 (C)

For ideal gasses (potential energy of the system taken as zero), we regard the internal energy as the kinetic energy of the molecules. It, therefore, changes only when the temperature of the system changes.
For practical systems, we can also associate a potential energy with a system which changes with a change in the state of the system. The internal energy of such practical systems can, therefore, change even when there is no change in their temperature. The changes, associated with a change of state of a solid into a liquid, a liquid into a gas, are examples of such situations.

A.7.1.8 (D)

The thermodynamic state is expressed in terms of the three variables (P, V, T). It is clear, therefore, that the thermodynamic state of a system would change if any one of its three state variables has a value different from its initial value.

A.7.1.9 (D)

The direction of heat flow is determined not by the heat content but by the temperature. The temperature of system A being more than that of either system B or system C, heat would flow from A to B as well as from A to C.

In case of system B and C, the direction of heat flow would be from B to C as B is at a higher temperature than C.

A.7.1.10 (D)

Internal energy is directly related to the temperature of the system.

Module 7.2 Thermodynamic Processes

A.7.2.1 (D)

A close look at the initial and final values of (P, V and T) reveals that the expansion

(i)in case (i), is an isothermal expansion

(ii)in case (ii), is an isobaric expansion

The P-V diagrams, corresponding to these two types of expansion, are the ones shown in Option (D).

A.7.2.2 (A)

The temperature, and therefore, the internal energy of a system remains constant in an isothermal change. The heat supplied is all used up in doing work against the surroundings.

A.7.2.3 (B)

For the process

$$P = aV^3 + bV^2 + cV + d$$

$$\frac{dP}{dV} = 3aV^2 + 2bV + c$$

For an adiabatic process

$$PV^{\gamma} = k \therefore \frac{dP}{dV}V^{r} + \gamma P\, V^{\gamma-1} = 0$$

Or $\frac{dP}{dV} = -\frac{\gamma\, P\, V^{\gamma-1}}{V^{\gamma}} = -\gamma\, P\, V^{-1} = -\frac{\gamma P}{V}$

$\therefore$ Ratio of the two slopes, at the point (P_0, V_0), is

$$\frac{(3aV_0^{\ 2}+2bV_0+c)V_0}{-\gamma P_0} \quad =-\frac{1}{\gamma P_0}\left[3aV_0^{\ 3}+2bV_0^2+cV_0\right]$$

A.7.2.4 (B)

The gas is undergoing an adiabatic change.

$$PV^{\gamma} = \text{constant}$$

$$\therefore P_2V_2^{\gamma} = P_1V_1^{\gamma}$$

$$\text{or } P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$P_1(2)^{\gamma}\left(\because V_2 = \frac{1}{2}V_1\right)$$

$$\therefore \frac{P_2}{P_1} = 2^{\gamma}$$

Hence,

For hydrogen, a diatomic gas, $\gamma = 5/3$

$$\therefore \frac{P_2}{P_1} = 2^{5/3}$$

A.7.2.5 (A)

For container A,

$$P_{2A}\frac{V}{4} = PV \qquad \text{(A.7.2.1)}$$

For container B

$$PV^{\gamma} = P_{2B}(V/8)^{\gamma} \tag{A.7.2.2}$$

Dividing Eq(A.7.2.2) by Eq(A.7.2.1) we will have

$$\frac{P_{2B}}{P_{2A}} = 2^{11/5}$$

A.7.2.6 (A)

We know that

$$W = \int_{P_1}^{P_2} PdV$$

The process being adiabatic, we can write

$$P(V + \Delta V)^{\gamma} = (P + \Delta P)V^{\gamma}$$

$$\therefore PV^{\gamma}\left(1 + \frac{\Delta V}{V}\right)^{\gamma} = PV^{\gamma}\left(1 + \frac{\Delta P}{P}\right)$$

$$\therefore \left(1 + \frac{\Delta P}{P}\right) \simeq \left(1 + \gamma\frac{\Delta V}{V} + ...\right)$$

or, $\frac{\Delta P}{P} \simeq \gamma\frac{\Delta V}{V}$

or, $\Delta V = \frac{V}{\gamma P}\Delta P$

$\therefore W = \int_{P_1}^{P_2} PdV = \int_{P_1}^{P_2} P\frac{V}{\gamma P}dP = \frac{V}{\gamma}\int_{P_1}^{P_2} dP = \frac{V}{\gamma}(P_2 - P_1) = \frac{3}{5}V(P_2 - P_1)(\therefore \gamma = 7/5$ s) for a diatomic gas.

A.7.2.7 (C)

Option (A.) is incorrect because an isotherm can intersect an adiabat.

Option (B.) is incorrect because the complete conversion of mechanical energy into heat is permissible.

Option (C.) is correct. In the cycle shown XY is an isotherm for which (as per the first law) $dQ = dW$ ($dU = 0$ for an isotherm). Also $dQ = 0$ for the parts YZ and ZX as they are both labeled as 'adiabatic'. Hence as per this cycle, we have $dQ = dW$, i.e., heat is getting completely converted into work. This, as we know, is NOT in accordance with the second law of thermodynamics.

Option(D.) is incorrect because here $U_X = U_Y (\neq U_Z)$. There is no change in internal energy during an isothermal change. Internal energy, however, changes during an adiabatic change).

A.7.2.8 (C)

for an adiabatic change

$$PV^{\gamma} = \text{constant}$$

$\therefore \frac{nRT}{V}.V^{\gamma} = \text{constant}$ Also, for a perfect gas, $PV = nRT = \text{constant}$

or $TV^{\gamma-1} = \text{constant}$

$\therefore T'(32V)^{\frac{7}{5}-1} = T\ (V)^{\left(\frac{7}{5}-1\right)} (\gamma = \frac{7}{5})$ for a diatomic gas

$$\text{Or } \frac{T'}{T} = \frac{V^{\frac{2}{5}}}{(32V)^{\frac{2}{5}}} = \frac{1}{(32)^{\frac{2}{5}}} = \frac{1}{4} = 2^{-2}$$

A.7.2.9 (A)

Net work done = $W_{AB} + W_{BC} + W_{CD} + W_{DA}$

Work done, during the adiabatic expansion from A to B, is (Here $PV^{\gamma} = K$)

$$W_{AB} = \int_{v_A}^{v_B} PdV = \int_{v_A}^{v_B} \frac{K}{V^{\gamma}} dV = \frac{K}{(1-\gamma)}\left[V_B^{1-\gamma} - V_A^{1-\gamma}\right] = \frac{1}{(1-\gamma)}\left[P_B V_B - P_A V_A\right] \quad \left[\because K = P_B V_B^{\gamma} \qquad = P_A V_A^{\gamma}\right]$$

Similarly $W_{CD} = \frac{1}{(1-\gamma)}\left[P_D V_A - P_C P_B\right]$

Also $\quad W_{BC} = W_{DA} = 0$ (these are isochoric changes)

Net work done= $W_{AB} = W_{CD} = \frac{1}{(1-\gamma)}\left[\left(P_B - P_C\right)V_B + \left(P_D - P_A\right)V_A\right]$

A.7.2.10 (C)

The given (*P-V*) diagram (and its mathematical expression) show that the gas is undergoing an isothermal expansion.

Hence the *T-P* diagram must be a straight line parallel to the *P*-axis. Further, as per the given diagram, the pressure, as point(1), is more than that at point (2). Hence the correct line corresponds to option (C). (The line, in Option (D), shows the pressure values in the reverse (incorrect) order) .

A.7.2.11 (D)

The given (*P-V*) diagram, and (its mathematical form) show that the gas is undergoing an isothermal compression.

Hence the (*T-V*) diagram must be a straight line parallel to the *V*-axis. In this diagram, the value of *V*, at point (1), must be more than that at point (2). The diagram, given in Option(D), is in accord with both these requirements.

Module 7.3 First Law of Thermodynamics

A.7.3.1 (C)

By the first law of thermodynamics

$dQ = dU + dW$

Or $dU = dQ - dW$

$\therefore$ $(U_Y - U_X)$(for path 1) = 1000 J $- W_1$ and $(U_Y - U_X)$(for path 2) = $Q - W_2$

$\therefore$ 1000 J$- W = Q - W_2$

or $Q = (1000 - W_1 + W_2) = [1000 - (W_1 - W_2)]$ J $= (1000 - 100)$ J $= 900$ J

A.7.3.2 (C)

In the usual statement of the first law of thermodynamics, it is assumed that the heat supplied to the system is used

(i)partly in increasing the internal energy and, therefore, the temperature of the system, and

(ii)partly in doing external work, i.e. work done by the system (on the surroundings).

Thus dU is (+ve) when the temperature of the system increases and dW is (+ve) when work is done by the system.

A.7.3.3 (B)

(i)During an isothermal change, the temperature and therefore, the internal energy of the gas remains unchanged. Hence $dU = 0$

(ii)During an isochoric change, the volume of the gas remains unchanged. Hence there is no work done by/on the gas. Hence $dW = 0$.

A.7.3.4 (B)

We have by the first law of thermodynamics,

$dQ = dU + dW$

$dU = dQ - dW$

In **case (i)**, $dW = 75$ joule per second and $dU = (100 - 75)$ joule/sec $= 25$ joule per second

and in **case (ii)**, $dW = -75$ joule per second

and $dU = (100 - (-75)$ joule per second $= 175$ joule per second.

A.7.3.5(A)

We have $dQ = dU + dW$

In the first case, $dQ = 0$. Hence

$dU = - dW = - (-22.3 \text{ J}) = 22.3 \text{ J}$

In the second case,

$dQ = 39.2 \text{ J}$

$dW = dQ - dU = (39.2 - 22.3) \text{ J} = 16.9 \text{ J}$

(dU is same as the initial and the final states are the same in this case also)

A.7.3.6 (C)

The given value of C_P, for the gas, indicates that it is a monatomic gas. Hence (from the given names) the permissible name, for this gas, could only be helium or neon.

Also $C_V = C_P - R = \frac{3}{2}R$

As per the first law of thermodynamics. We have $dQ = dU + dW$

During a constant volume (isochoric) process,

$dW\left(= \int PdV\right)$ is zero as there is no change in the volume of the gas. Also

$$dU = \eta C_V dT = \frac{3}{2}\eta RdT$$

$$\therefore dQ = \frac{3}{2}\eta RdT$$

Hence the rise in temperature of the gas is

$$dT = \frac{Q}{(\frac{3}{2\eta R})} = \frac{2}{3}\ \frac{Q}{\eta R}.$$

A.7.3.7 (C)

It is only the internal energy (of a system) that depends only on the state of the system and not on the way that state has arrived. Thus ΔU remains the same irrespective of the way the transition (between two given states) is brought about. This is not true for ΔH and ΔW; they depend on the 'path followed' in going from the initial to the final state.

A.7.3.8 (C)

Using the gas equation, we find that the temperature at L should be given by

$(3P_o)\ (3V_o) = 1.R.T_L$ or $T_L = \frac{9P_0V_0}{R}$

For the point K, we have

$$P_oV_o = 1.R.T_K$$

Or $T_K = \frac{P_o V_o}{R}$

$\therefore T_L = 9T_K = 9\ T_o$

Now $dQ = dU + dW$

Here $dU = c_V(9T_0 - T_0) = 8\ C_V T_0 = 8\left(\frac{3}{2}R\right)T_0$

$= 12RT_0 (\because C_V = \frac{3}{2}R)$ for monoatomic gas

A.7.3.9 (D)

The internal energy of a gas, in a given state, is a characteristic of that state and does not depend on the way that state has been arrived at. The value of U_B (and U_A) would, therefore , be the same for all the four cases and hence

$\Delta U = U_B - U_A$

Would have the same value for all the four cases.

A.7.3.10 (C)

The work done by gas $\left(= \int_A^B PdV\right)$ depends on the path, followed by it, in going from its initial state to its final state. It therefore, equals the area under the corresponding P-V diagram.

We see, from the given diagram, that the area, under the curve, for path y is greater than that under 'path' x, which in turn, is greater than that under 'path' z. hence $W_y > W_x > W_z$

A.7.3.11 (C)

When a gas undergoes an adiabatic change, the heat added to (or taken out of) it is zero. By first law,

$dQ = dU + dW = 0$

$$\therefore\ dU = -\,dW$$

Here dU is positive as the temperature, and, therefore, the internal energy of the gas have increased. This means that dW must be taken with a negative sign. Hence work has been done ON the gas. The gas must have, therefore, been (adiabatically) compressed.

A.7.4.1 (D)

We have

$$9 = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

$$\therefore 9 = \frac{T_2}{300 - T_2}$$

$$\therefore 2700 = 10\, T_2$$

Or $T_2 = 270$ Kelvin

$\therefore$ Temperature inside the refrigerator $= -3°C$

A.7.4.2 (B)

The coefficient of performance, of the ideal refrigerator, working between a 'sink' temperature of T_2 kelvin and a 'source' temperature of T_1 kelvin, is $\frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2} = \frac{270}{300 - 270} = 9$

Hence, in this case, the coefficient of performance being 50%, the effective coefficient of performance, is $0.5 \times 9 = 4.5$

Hence $\frac{Q_2}{W} = 4.5$

Or $Q_2 = 1 \times 4.5$ kW $= 4.5$ kW

$\therefore$ Heat taken out of the refrigerator per second is (4.5 kJ/sec)

A.7.4.3 (A)

For section AB, $dQ = dU = \frac{3}{2}R\left(T_B - T_A\right) = \frac{3}{2}V_A\left(P_B - P_A\right)$

For section BC $\mathrm{d}Q = \mathrm{d}U + \mathrm{d}W$

$$= \frac{3}{2}P_B\left(V_C - V_B\right) + P_B\left(V_C - V_B\right) = \frac{5}{2}P_B\left(V_C - V_B\right)$$

For section CD, $dQ = O$

For section DA, $dQ = dU + dW$

$$= \frac{3}{2}P_A\left(V_A - V_D\right) + P_A\left(V_A - V_D\right) = \frac{5}{2}P_A\left(V_A - V_D\right)$$

A.7.4.4 (B)

Coefficient of performance

$$= \frac{T_2}{T_1 - T_2} = \frac{273+9}{(273+36)-(273+9)} = \frac{282}{27} = 10.4$$

A.7.4.5 (B)

Coefficient of performance $= \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$

$$\therefore\ 5 = \frac{(273+\theta)}{(273+27)-(273+\theta)} = \frac{273+\theta}{27-\theta}$$

$\therefore$ $135 - 5\,\theta = 273 + \theta$ or $6\theta = -138$ ºC $\therefore$ $\theta = -23$ºC

A.7.4.6 (C)

For the Carnot cycle

$$\frac{H_1}{H_2} = \frac{T_1}{T_2}$$

and $H_1 - H_2 = W$

$$\therefore \frac{H_1}{H_2} = \frac{500}{300} = \frac{5}{3}$$

and $H_1 - H_2 = 1$ kJ

$$\therefore H_1 - \frac{3}{5}H_1 = 1 \text{ k J}$$

$$\text{or } H_1 = \frac{1 \times 5}{2} \text{ kJ} \quad = 2.5 \text{ kJ}$$

$$\therefore H_2 = \frac{3}{5}H_1 = 1.5 \text{ kJ}$$

A.7.4.7 (B)

efficiency of the heat engine = $\frac{\text{Work done}}{\text{Heat absorbed}}$

$$n_1 = \frac{1.4 \times 10^3}{1 \times 4.2 \times 10^3} = \frac{1}{3}$$

For the ideal heat engine,

$$\text{Efficiency} = \eta = \frac{T_1 - T_2}{T_1} = \frac{600-300}{600} = \frac{1}{2}$$

$$\therefore \frac{\eta_1}{\eta_2} = \frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$$

A.7.4.8 (A)

This setup corresponds to a refrigerator in which the system is taking heat from the sink, having work done on it, and transforming heat is the source. For this case, both Q_1 and Q_2 are positive with $Q_1 > Q_2$

A.7.4.9 (B)

coefficient of performance $= \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$

$\therefore 1 = \frac{T_2}{T_1 - T_2}$

or $T_1 = 2T_2$

or $(t_1 + 273) = 2\,(t_2 + 273)$

$\therefore t_1 = (2t_2 + 273)$

A.7.4.10 (C)

efficiency of an ideal heat engine $= \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$

$$\therefore 25\,\% = \frac{1}{4} = \frac{T_1 - T_2}{T_1}$$

$$\text{Or } 4T_1 - 4T_2 = T_1$$

$$\therefore T_1 = \frac{4}{3} T_2$$

$$\text{Or } (t_1 + 273) = \frac{4}{3}(t_2 + 273) = \frac{4}{3} t_2 + 364$$

$$t_1 = \frac{4}{3} t_2 + 91$$

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Module 8.1 Simple Harmonic Motion

Q.8.1.1

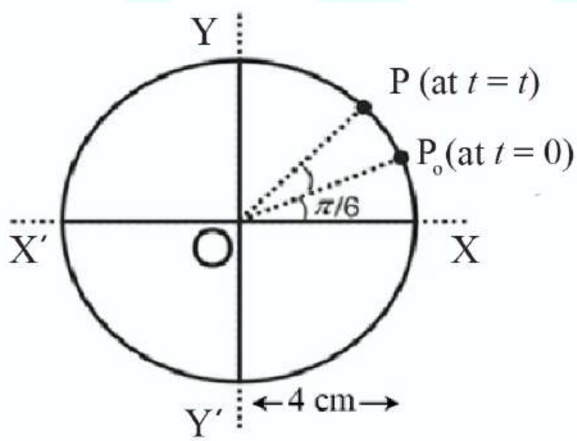


Fig.Q.8.1.1

Fig.Q.8.1.1 shows a particle moving with uniform speed along a circle of radius 4 cm in an anticlockwise direction with a time period 0.2 s. P_0 is the position of the particle at $t = 0$, and P is the position of the particle at $t = t$ as shown, consider the foot of the perpendicular from P on the diameter Y′OY. If it is given by y (in cm). Then ,

(A) $y = 4 \sin \left[10\pi t + \frac{\pi}{6}\right]$

(B) $y = 4 \sin \left[10\pi t - \frac{\pi}{6}\right]$

(C) $y = 4\cos\left[10\pi t + \frac{\pi}{6}\right]$

(D) $y = 4\cos\left[10\pi t + \frac{\pi}{6}\right]$

Q.8.1.2. The equation describing SHM of a particle along X-axis with symbols having their usual meaning, is $x = 4\sin\left[100\pi t + \frac{\pi}{6}\right]$

The equation can be rewritten as

(A) $x = 2\sqrt{3}\sin(100\pi t) + 2\cos(100\pi t)$

(B) $x = 2\sqrt{3}\sin(100\pi t) - 2\cos(100\pi t)$

(C) $x = 2\sin(100\pi t) + 2\sqrt{3}\cos(100\pi t)$

(D) $x = 2\sqrt{3}\cos(100\pi t) - 2\sin(100\pi t)$

Q.8.1.3. The magnitude (v) of the velocity of a particle moving along the x-axis is expressed as a function of x, as under

$v^2 = 24x - 2x^2 - 70$ (all quantities are cgs units)

Then , which among the following statements is/are correct about the motion?

I. it is anharmonic with maximum velocity at $x = 12$

II. it is anharmonic with minimum velocity at $x = 12$

III. it is simple harmonic with amplitude equal to 1 cm

IV. it is a simple harmonic with a time period equal to π second.

(A) only I

(B) only II

(C) only III

(D) III and IV

Q.8.1.4 A particle of mass m, moving along X-axis has instantaneous acceleration; is given by

$$a = -[4x+1]$$

x is the instantaneous position coordinate of the particle. The motion of a particle is an SHM with time period T and mean position at x_0 , where

(A) $T = \pi, \quad x_0 = 0$

(B) $T = 2\pi, \quad x_0 = 0$

(C) $T = 2\pi, \quad x_0 = 0.25$

(D) $T = \pi, \quad x_0 = -0.25$

Q.8.1.5 The instantaneous displacement, x, (in cm) of a particle moving along X-axis is

$$x = 8 \sin 10\pi t \cos 10\pi t$$

Which of the following statements is correct?

(A) Motion is not simple harmonic

(B) Motion is simple harmonic, with $A = 8$ cm, $\nu = 5$ Hz

(C) Motion is simple harmonic , with $A = 4$ cm, $\nu = 10$ Hz

(D) Motion is simple harmonic , with $A = 8$ cm, $\nu = 10$ Hz

Q.8.1.6 The differential equation of motion of a particle of mass 200 g, executing SHM, along X-axis is

$\frac{d^2x}{dt^2} = -\ 9x$, x is in meters; t is in seconds.

The time period T; and the spring constant k are

(A) $T = \frac{2\pi}{3} s$; $k = 1.8$ Nm^{-1}

(B) $T = \frac{2\pi}{9} s$; $k = 0.18$ Nm^{-1}

(C) $T = \frac{2\pi}{3} s$; $k = 1.8$ dyne cm^{-1}

(D) $T = \frac{2\pi}{3} s$; $k = 1.8 \times 10^3$ Nm^{-1}

Q.8.1.7 A particle of mass m executes SHM of time period T, under a restoring force of spring constant k. The mass is decreased by 25% and k is increased by 25%. The time period of oscillation is T′, $\frac{T'}{T}$ is

(A) $\sqrt{\frac{3}{5}}$

(B) 1

(C) $\sqrt{\frac{5}{3}}$

(D) $\frac{1}{\sqrt{2}}$

Q.8.1.8 A particle executes SHM of amplitude A and time period T along the X-axis. At $t = 0$; the particle is in its extreme position along the positive direction of displacement. The particle is in position P and Q at $t = \frac{T}{12}$ and $t = \frac{T}{8}$ respectively. The percentage change in displacement, as a particle moves from P to Q, is nearly

(A) 16%

(B) – 18.4%

(C) + 18.4%

(D) – 24%

Q.8.1.9 Two particles execute SHM of same amplitude A and same frequency ν along the X-axis. The maximum separation between the two particles during their motion is $\sqrt{3}\,A$. The initial phase difference, ϕ_0 , between the two SHM's is

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$

Q.8.1.10 The momentum vs. position curve for a particle of mass m executing SHM, of force constant, k, is a circle .Then the numerical value of m is equal to

(A) k

(B) $\frac{1}{k}$

(C) k^2

(D) $\frac{1}{k^2}$

Module 8.2 Characteristics of Simple Harmonic Motion

Q.8.2.1 A particle executes SHM of amplitude A, angular frequency ω . v is the instantaneous speed of the particle when the instantaneous displacement is 50% of A. v_m is the velocity amplitude of the motion. Δv , the difference of v and v_m, expressed as a fraction of v_m is

(A) $\left(1-\frac{\sqrt{3}}{2}\right)$

(B) 0.5

(C) $\left(1-\frac{1}{\sqrt{2}}\right)$

(D) $\left(\frac{\sqrt{3}}{2}-1\right)$

Q.8.2.2 For a particle moving along the X-axis, x and v denote the instantaneous displacement and the instantaneous speed respectively. Given

$$\frac{x^2}{4}+\frac{v^2}{4\pi^2}=1$$

x is in cm and time in second. If A is the amplitude in cm, and T, the time period in sec, then which among the following statements is correct?

(A) The motion is not simple harmonic.

(B) The motion is simple harmonic, with $A = 2$, $T = 2$

(C) The motion is simple harmonic with $A = 4$, $T = \frac{1}{2\pi}$

(D) The motion is simple harmonic with $A = 2$, $T = 1$

Q.8.2.3 The amplitude and the time period of a simple harmonic oscillator of mass 200 g, are 5 cm and 2 sec respectively. When the oscillator is at a distance x cm from the mean position its kinetic energy is equal to 1.6 mJ; the value(s) of x is/are (given $\pi^2 = 10$).

(A) ± 2

(B) Only 2

(C) Only – 3

(D) ± 3

Q.8.2.4 A particle executing SHM has a speed α when its displacement from mean position is β. If the magnitude of displacement increases by 25% , the magnitude of the speed decreases by 50%. The amplitude (A) and time-period (T) of SHM is

(A) $A = \frac{\beta}{2\alpha}$; $T = \frac{\alpha}{\beta}$

(B) $A = \frac{\sqrt{5}}{2}\beta$; $T = \pi\left(\frac{\beta}{\alpha}\right)$

(C) $A = \frac{\sqrt{7}}{2}\beta$; $T = \sqrt{3}\pi\left(\frac{\beta}{\alpha}\right)$

(D) $A = \frac{2}{\sqrt{7}}\beta$; $T = \sqrt{3}\pi\left(\frac{\alpha}{\beta}\right)$

Q.8.2.5 The instantaneous acceleration (a) versus the instantaneous displacement (x) graph, for two SHM's I and II is shown in Fig.Q.8.2.1.

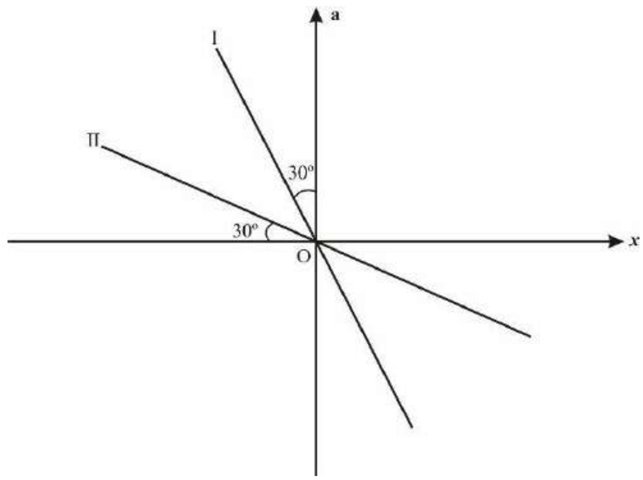


Fig.Q.8.2.1

The ratio of the time periods T_1 and T_2 in the two cases is

(A) 1:1

(B) $1:\sqrt{3}$

(C) $\sqrt{3}:1$

(D) 2:1

Q.8.2.6 A particle executes SHM of amplitude A and total energy E. When the instantaneous displacement of the particle is $\pm\frac{A}{\sqrt{2}}$, K and U denote the kinetic and potential energy of the particle. What is (i) $\frac{U}{K}$ and (ii) $\frac{E}{U}$?

(A) $\frac{U}{K} = 1; \frac{E}{U} = 2$

(B) $\frac{U}{K} = 1; \frac{E}{U} = 1$

(C) $\frac{U}{K} = 2; \frac{E}{U} = \frac{3}{2}$

(D) $\frac{U}{K} = \frac{1}{2}; \frac{E}{U} = \frac{2}{3}$

Q.8.2.7_A particle of mass 25 g executes SHM under a force of spring constant 10^{-3} Ncm^{-1}. The average kinetic energy over one cycle of changes is 16 mJ. The amplitude(A); and angular frequency (ω) of SHM are

(A) $A = 0.4\sqrt{2}\, m$; $\omega = 1$ rad s^{-1}

(B) $A = 0.8$ m ; $\omega = 2$ rad s^{-1}

(C) $A = 0.8$ m ; $\omega = 1$ rad s^{-1}

(D) $A = 0.4$m ; $\omega = 1$ rad s^{-1}

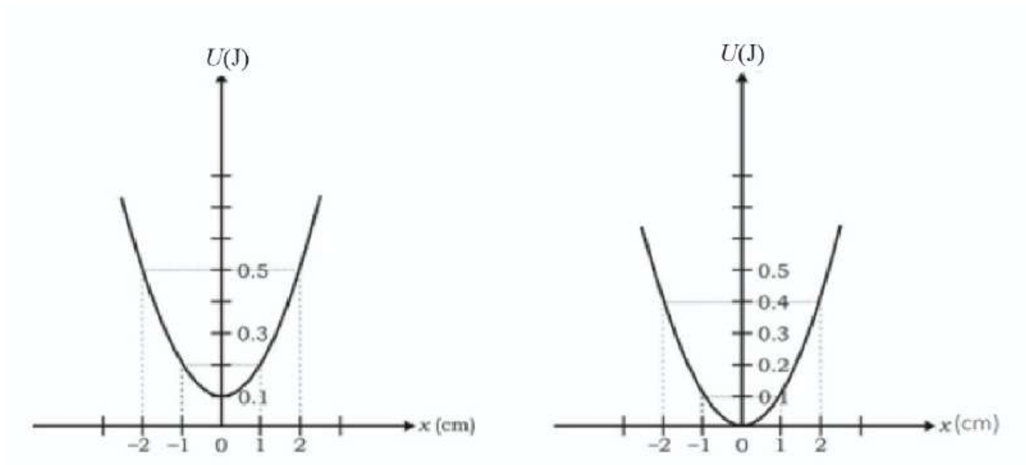
Q.8.2.8 The potential energy(U) versus the displacement(x) graph for two SHMs is as shown in Fig.Q.8.2.2(a) and Fig.Q.8.2.2(b). The ratio of the spring constant in Fig.Q.8.2.2(a) and Fig.Q.8.2.2(b) is

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) 1 : 4



(a) (b)

Fig.Q.8.2.2

Q.8.2.9 $U = \alpha + \beta x^2$ is the relation between the instantaneous potential energy(U) and instantaneous displacement(x) of a particle of mass m. α and β are constants. Which of the following statements is correct?

(A) Motion is periodic but not simple harmonic

(B) Motion is a simple harmonic of time-period $2\pi\sqrt{\frac{m}{\beta}}$

(C) Motion is a simple harmonic of time-period $2\pi\sqrt{\frac{m}{2\beta}}$

(D) Motion is neither periodic nor simple harmonic

Q.8.2.10 The diagram of a simple pendulum executing SHM is shown in Fig.Q.8.2.3. The symbols have their usual meaning. The energy conservation equation corresponding to angular displacement θ reduces to-

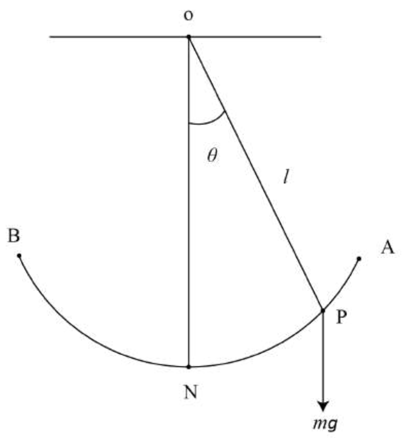


Fig.Q.8.2.3

(A) $\frac{1}{2}l\left(\frac{d\theta}{dt}\right)^2 + g\cos\theta = \text{a constant}$

(B) $\frac{1}{2}l\left(\frac{d\theta}{dt}\right)^2 - g\cos\theta = \text{a constant}$

(C) $l\left(\frac{d\theta}{dt}\right)^2 + g\sin\theta = \text{a constant}$

(D) $l\left(\frac{d\theta}{dt}\right)^2 - g\sin\theta = \text{a constant}$

Module 8.3 Examples of Simple Harmonic Motion

Q.8.3.1 A mass M attached to a spring of spring constant k_1 executes vertical oscillations of time period T_1. The mass is reduced by 25% and attached to another spring of spring constant k_2. The time period of vertical oscillations is 50% more than T_1. $\frac{k_2}{k_1}$ is

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $\frac{9}{4}$

(D) $\frac{4}{9}$

Q.8.3.2 A mass M attached to the free end of a vertical spring executes SHM of time-period T. The mass is immersed completely in a non-viscous liquid as shown in Fig.Q.8.3.1(b). What change ; if any, is observed in the oscillations of mass M ? The mass remains inside liquid throughout the motion.

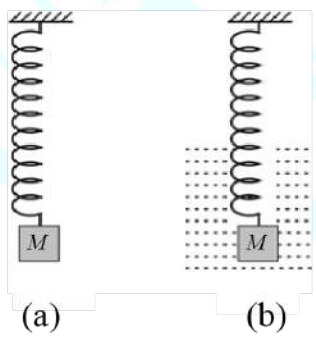


Fig.Q.8.3.1

(A) There is no change

(B) Time period increases.

(C) The mean (or equilibrium) position moves upwards. Time period remains the same.

(D) The mean (or equilibrium) position moves downwards. Time period remains the same.

Q.8.3.3 A hollow container of mass m_1 is filled completely with a liquid of mass m_2. The arrangement attached to the free end of a vertical spring of spring constant k, executes SHM (Fig.Q.8.3.2). There is a small hole in the container through which liquid drips out at a constant rate α. The graph showing the variation of (T^2) with time(t) is

[T = instantaneous time period of the system]

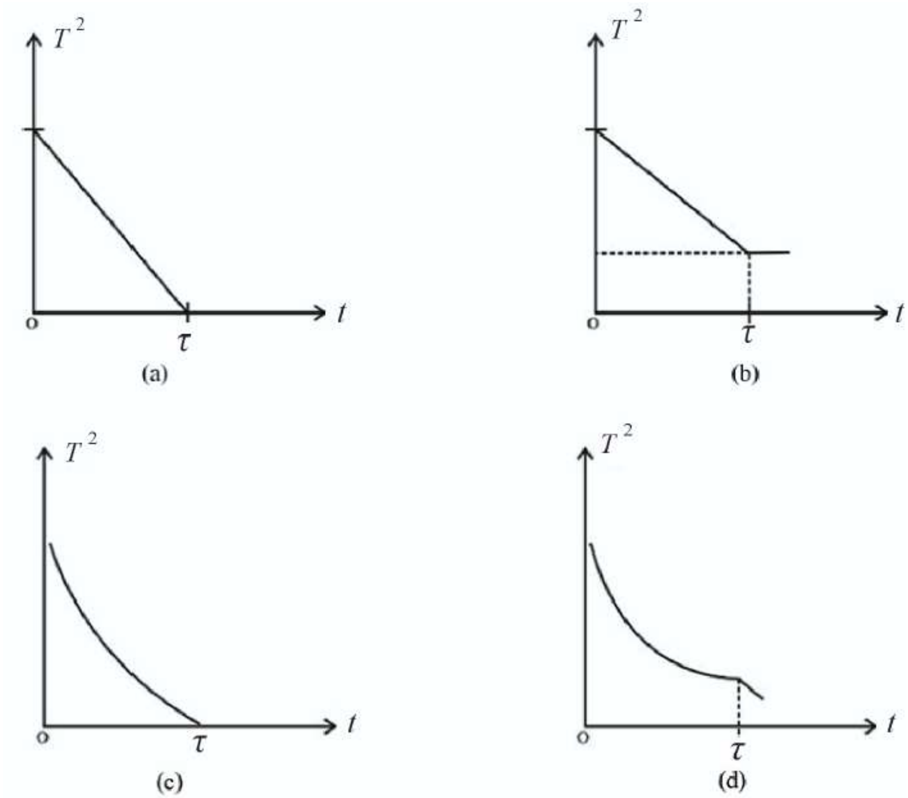


Fig.Q.8.3.2

Q.8.3.4 A mass M attached to a spring S_1, (spring constant = k_1) ; and S_2 (spring constant = k_2) one by one executes horizontal oscillations of time period T_1 and T_2 respectively. The same mass attached to both S_1 and S_2 simultaneously as shown in Fig.Q.8.3.3; executes oscillations of time-period T Which of the following is correct?

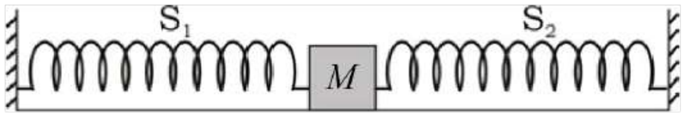


Fig.Q.8.3.3

(A) $T = T_1 + T_2$

(B) $T^2 = T_1^2 + T_2^2$

(C) $\frac{1}{T_2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$

(D) $\frac{2}{T} = \frac{1}{T_1} + \frac{1}{T_2}$

Q.8.3.5 For the arrangement shown in Fig.Q.8.3.4; the time-period of oscillations of mass *M* is

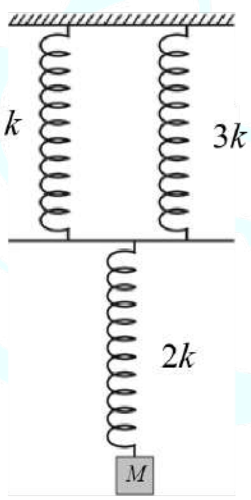


Fig.Q.8.3.4

(A) $2\pi\sqrt{\frac{M}{6k}}$

(B) $2\pi\sqrt{\frac{3M}{k}}$

(C) $\pi\sqrt{\frac{3M}{k}}$

(D) $2\pi\sqrt{\frac{2M}{3k}}$

Q.8.3.6 The time period, *T*; of the vertical oscillations of a cylinder of radius *R*; length *L*: mass *M* and density ρ ; immersed in a liquid of density σ is

$$T = 2\pi\sqrt{\frac{M}{\pi R^2 \sigma g}}$$

The radius of the cylinder is doubled without any other change. The time-period of oscillations of cylinder now is

(A) T

(B) $2T$

(C) $\frac{T}{2}$

(D) $\frac{T}{\sqrt{2}}$

Q.8.3.7 A U-tube contains two immiscible liquids. In equilibrium the length of the two liquid columns in the two limbs of the tube is L_1 and L_2 as shown in Fig.Q.8.3.5 ρ_1 and ρ_2 are the densities of the two liquids. For small displacements of the liquid columns, the time period, *T*, of oscillations is

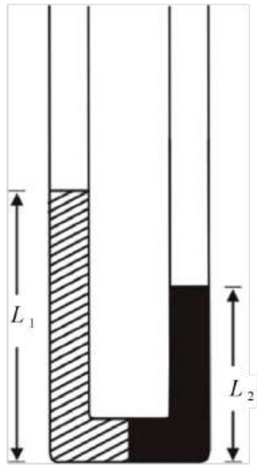


Fig.Q.8.3.5

(A) $2\pi\sqrt{\frac{L_1+L_2}{2(\rho_1+\rho_2)g}}$

(B) $2\pi\sqrt{\frac{L_1+L_2}{(\rho_1+\rho_2)g}}$

(C) $2\pi\sqrt{\frac{L_1\rho_2+L_2\rho_1}{(\rho_1+\rho_2)g}}$

(D) $2\pi\sqrt{\frac{2L_1\rho_1}{(\rho_1+\rho_2)g}}$

Q.8.3.8 A vertical U-tube has limbs of cross-sectional area (a). A liquid of density (ρ) fills the tube up to a length (L), in equilibrium. The time period of oscillations of liquid is (T). The area of cross-section of each limb of the U-tube is doubled. It is filled up to the same length (L), by a liquid of density (2ρ) in equilibrium. The time-period of oscillation of liquid now is (T_1). What is ($\frac{T_1}{T}$)?

[Given T = Time period of oscillations

$$= 2\pi\sqrt{\frac{M}{2A\rho g}}$$

M = mass of liquid in U-tube]

(A) 0.5

(B) 0.707

(C) 1

(D) 1.414

Q.8.3.9 A second pendulum is suspended from the ceiling of an accelerating lift. The time-period is observed to increase by 50% of its earlier value. The acceleration of lift is

(A) $\frac{2g}{3}$; upwards

(B) $\frac{4g}{9}$; downwards

(C) $\frac{5g}{9}$; downwards

(D) $\frac{5g}{9}$; upwards

Q.8.3.10 A second pendulum has a bob of a material of density ρ. The bob is immersed completely in a non-viscous liquid of density $\sigma = \frac{\rho}{2}$. The time period of oscillations is

(A) 2 s

(B) $2\sqrt{2}$ s

(C) $\sqrt{2}$ s

(D) 1 s

Q.8.3.11 A spherical shell filled completely with a liquid is used as the bob of a simple pendulum. As the pendulum starts oscillating at $t = 0$; the liquid drips out at a constant rate; till the shell is empty. The correct statement, describing time period of pendulum is:

(A) Remains the same throughout

(B) Increases

(C) Decreases

(D) First increases acquire a maximum value, then decreases and finally is the same as at $t = 0$.

Module 8.4 Free, Damped and Forced Oscillations

Q.8.4.1 What is the unit of b used in the differential equation of motion given by

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega^2 x = 0$$

(A) kgs^{-1}
(B) s^{-1}
(C) kgm^{-1}
(D) $kg\ ms^{-1}$

Q.8.4.2 What are the Order (O) and Degree (D) of the differential equation of damped harmonic motion?

(A) O-2, D-2
(B) O-2, D-1
(C) O-1, D-2
(D) O-1, D-1

Common information for Q.8.4.3, Q.8.4.4, Q.8.4.5.

For a damped harmonic oscillator of mass 0.28 kg, the force constant is 80 Nm^{-1} and the damping constant 0.25 s^{-1}.

Q.8.4.3 If instead of the angular frequency of damped harmonic motion, the same for undamped vibration is determined, what will be the percentage error (correct up to three decimal places)?

(A) + 0.023 %
(B) + 1.128 %
(C) – 0. 012 %

(D) – 1. 114 %

Q.8.4.4 In what time (correct up to three decimal places in seconds) does the amplitude reach to half its value?

(A) 2.773

(B) 1.387

(C) 0.694

(D) 0.347

Q.8.4.5 In what time (correct up to three decimal places in seconds) does the mechanical energy get reduced to half its original value?

(A) 5.546

(B) 2.773

(C) 1.387

(D) 0.694

Q.8.4.6 Resonant vibration is also called _________ vibration.

(A) menacing

(B) threatening

(C) sympathetic

(D) reverberation

Q.8.4.7 For which among the following, the primary reason is NOT 'Resonance'?

(A) Reverberation in an auditorium.

(B) Soldiers are being asked to break steps while marching on a suspension bridge.

(C) Collapsing of some buildings far away from the epicenter of an earthquake, while not much damage is caused to many structures located close to it.

(D) Violent flapping of aircraft wings corresponds to some engine speed.

Q.8.4.8 For a lowly damped oscillator resonant frequency(p_r) is

(A) nearly equal to the natural frequency(ω) of an oscillator

(B) greater than natural frequency of an oscillator

(C) less than natural frequency of an oscillator

(D) independent of natural frequency of an oscillator

Common information for Q.8.4.9 and Q.8.4.10

A system having a natural angular frequency of oscillation, ω is subject to forced oscillation by the application of an external periodic force proportional to sin pt, in presence of damping, characterized by the constant b which is equal to half the damping constant per unit mass and unit velocity of the oscillator.

Q.8.4.9 On which among the following does the amplitude of forced oscillation depend?

(A) ω, but not on p and b

(B) ω and p, but not on b

(C) b, but not on ω and p

(D) b,ω and p

Q.8.4.10 If the forced oscillation lags behind the free oscillation by 45^0, then the value of p is given by

(A) $b+\sqrt{(\omega^2+b^2)}$

(B) $\sqrt{(\omega^2+b^2)}-b$

(C) $\sqrt{(\omega^2+b^2)}-\omega$

(D) $\omega+\sqrt{(\omega^2+b^2)}$

ANSWERS

Module 8.1 Simple Harmonic Motion

A.8.1.1 (A)

ω = Angular speed of P along circle of reference $= \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\,\pi$ s^{-1}

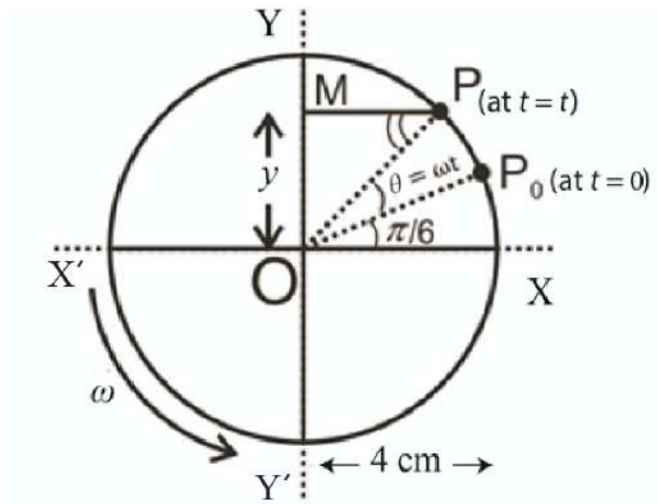


Fig.A.8.1.1

M is the foot of the perpendicular from P on Y'OY.

A = Amplitude of motion of M = 4 cm

$\angle$OPM = $\angle$OPM = $\omega t + \frac{\pi}{6} = 10\pi t + \frac{\pi}{6}$

From $\angle$OPM; y = OM = instantaneous displacement of M

= OP sin$\angle$OPM = 4sin[10πt + $\frac{\pi}{6}$]

A.8.1.2 (A.)

(i) Let us $2\sqrt{3} = A\cos\,\phi$; $2 = A\,\sin\,\phi$

$\left(2\sqrt{3}\right)^2 + (2)^2 = A^2$ or $A = 4$

$\tan\phi = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} \quad \phi = \frac{\pi}{6}$

Given equation is rewritten as

$x = 4 \sin(100\pi t)\cos(\frac{\pi}{6}) + 4\cos(100\pi t)\sin(\frac{\pi}{6})$

(ii) In the same manner ; we can rewrite

$x = 4\sin\left[100\pi t - \frac{\pi}{6}\right]$

(iii) Put $2 = A\sin\phi$; $2\sqrt{3} = A\cos\phi$

$\therefore 4 = A ; \phi = \frac{\pi}{6}$

and $x = 4\cos(100\pi t - \frac{\pi}{6})$

(iv) Can be rewritten as

$x = 4\cos(100\pi t + \frac{\pi}{6})$

A.8.1.3 (C)

$$v^2 = 24x - 2x^2 - 70 \qquad (8.1.1)$$

$$\therefore \frac{d}{dx}\left(v^2\right) = 2v\frac{dv}{dx} = 24 - 4x = -4(x - 6)$$

$$\therefore v\frac{dv}{dx} = -2(x - 6) \qquad (8.1.2)$$

$$v\frac{dv}{dx} = \frac{dx}{dt}\frac{dv}{dx} = \frac{dv}{dt} = \text{acceleration} = \frac{d^2x}{dt^2}$$

Let $z = x - 6$, then $\frac{d^2z}{dt^2} = \frac{d^2x}{dt^2}$

So, from Eq(8.1.2). $\frac{d^2z}{dt^2} = -2z$, which is the differential equation of SHM, with $\omega^2 = 2$

So, $\omega = \sqrt{2}$ s^{-1}

$T = \frac{2\pi}{\sqrt{2}} = \pi\sqrt{2}$ s

$v^2 = -2(x^2 - 12x + 35)$

$= -2(x - 5)(x - 7)$

So, $v = 0$ $x = 5$, $x = 7$. which are the extremities of the motion. Hence,

amp $= \frac{1}{2}(7 - 5) = \frac{1}{2}(2) = 1$ cm.

A.8.1.4 (D)

From the given condition , $\frac{d^2x}{dt^2} = -4\left(x + \frac{1}{4}\right)$

Let $z = x + \frac{1}{4}$, $\frac{dz^2}{dt} = \frac{d^2x}{dt^2}$

$\therefore \frac{d^2z}{dt^2} = -4z$

$\therefore \frac{d^2z}{dt^2} + 4z = 0$, which is the differential equation of SHM, with

$\omega^2 = 4$, $\omega = 2$ s^{-1}

T $= \frac{2\pi}{\omega} = \pi$ s

The mean position is z = 0, i.e. $x + \frac{1}{4} = 0$

So, $x_0 = -\frac{1}{4} = -0.25$

A.8.1.5 (C)

Given equation can be rewritten using trigonometric identity

$\sin 2\theta = 2\sin\theta\cos\theta$ as,

$$x = 4\sin 20\pi t \tag{8.1.3}$$

Comparing Eq(8.1.3) with the standard equation of SHM

$x = A\sin(\omega t + \phi_0)$, we have

A = 4 cm, $\omega = 20\pi$ rad s^{-1} or $\nu = \frac{20\pi}{2\pi} = 10$ Hz , $\phi_0 = 0$

A.8.1.6 (A)

Comparing $\frac{d^2x}{dt^2} = -9x$ with the standard equation of an SHM.

We get $\omega^2 = 9$, $\omega = 3$ s^{-1}

$T = \frac{2\pi}{\omega} = 2\pi/3$ s

Also, $\omega^2 = k/m$

$k = 1.8$ N/m

A.8.1.7 (A)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$m' = m - \frac{m}{4} = \frac{3}{4}m$, and $k' = k + \frac{k}{4} = \frac{5}{4}k$

$$\therefore T' = 2\pi\sqrt{\frac{m'}{k'}} = 2\pi\sqrt{\frac{3m\times4}{4\times5k}} = 2\pi\sqrt{\frac{3m}{5k}} = T\sqrt{\frac{3}{5}}$$

$$\therefore \frac{T'}{T} = \sqrt{\frac{3}{5}}.$$

A.8.1.8 (B)

At $t = 0$; $x = + A$. The equation of SHM is

$$x = A\cos\left(\frac{2\pi}{T}t\right)$$

x_1 = Displacement of particle at P

$$= A\cos\left(\frac{2\pi}{T}\times\frac{T}{12}\right) = A\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}A \backsimeq 0.867A$$

x_2 = Displacement of particle at Q

$$= A\cos\left(\frac{2\pi}{T}\times\frac{T}{8}\right) = A\cos\left(\frac{\pi}{4}\right) = \frac{A}{\sqrt{2}} \backsimeq 0.707A$$

Δx = change in displacement

$$= x_2 - x_1 = (0.707 - 0.867)A = -0.16A$$

% change in displacement

$$= -\left(\frac{0.16A}{0.867A}\right)\times 100 \simeq -18.4\%$$

A.8.1.9 (B)

Let the instantaneous displacement of the two SHM, be X_1 and X_2 ,we have

$$X_1 = A \sin 2\pi\nu t$$

$$X_2 = A \sin (2\pi\nu t + \phi_0)$$

here, ϕ_0 is the phase difference between the two motions.

x = instantaneous separation between the two particles = $x_2 - x_1$

$$= A[\sin\left(2\pi\nu t + \phi_0\right) - \sin 2\pi\nu t]$$

Using trigonometric identity: $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$,

x can be rewritten as

$$x = \left[2A\sin\left(\frac{\phi_0}{2}\right)\right]\cos\left(\omega t + \frac{\phi_0}{2}\right) \qquad \text{Where } \omega = 2\pi\nu$$

The maximum value of $\cos\left(\omega t + \frac{\phi_0}{2}\right)$ is +1.

Therefore $x_m = 2A\sin\left(\frac{\phi_0}{2}\right)$

Given $x_m = \sqrt{3}A$; therefore

$$\sqrt{3}A = 2A\sin\left(\frac{\phi_0}{2}\right) \text{ or } \frac{\sqrt{3}}{2} = \sin\left(\frac{\phi_0}{2}\right)$$

$$\text{or } \frac{\pi}{3} = \frac{\phi_0}{2}$$

$$\therefore \phi_0 = \frac{2\pi}{3}$$

A.8.1.10 (B)

With symbol having usual meaning, we have the energy equation

$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 =$ a constant = E, say

$\therefore \frac{p^2}{2mE} + \frac{x^2}{\frac{2E}{k}} = 1$, where $p = mv$

Since p-x curve is a circle , $2mE = \frac{2E}{k}$, $\quad \therefore m = \frac{1}{k}$

Module 8.2 Characteristics of Simple Harmonic Motion

A.8.2.1 (D)

For the particle executing SHM; the instantaneous speed v is

$$v = \omega\sqrt{A^2 - x^2} \quad \text{; Where } x \text{ is the instantaneous displacement.}$$

Given, $x = 50\%$ of $A = \frac{A}{2}$

Therefore

$$v = \omega\sqrt{A^2 - \left(\frac{A}{2}\right)^2} = \frac{\sqrt{3}}{2}A\omega = \frac{\sqrt{3}}{2}v_m$$

where $v_m = A\omega$ velocity amplitude. Obviously

$$\Delta v = v - v_m = \left(\frac{\sqrt{3}}{2} - 1\right)v_m$$

$$\text{or } \frac{\Delta v}{v_m} = \left(\frac{\sqrt{3}}{2} - 1\right)$$

A.8.2.2 (B)

For a particle executing SHM of amplitude A, angular frequency ω, the instantaneous speed, v, is

$$v = \omega\sqrt{A^2 - x^2} \qquad \text{or } \left(\frac{v}{\omega}\right)^2 = A^2 - x^2$$

$$\therefore \frac{v^2}{(A\omega)^2} + \frac{x^2}{A^2} = 1 \tag{8.2.1}$$

Comparing Eq(8.2.1) with the given equation

$A^2 = 4$ or $A = 2$ cm

$A\omega = 2\pi$ or $\omega = \pi$

$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$ s

A.8.2.3 (D)

Given $T = 2$ s; $A = 5$ cm $= 5 \times 10^{-2}$ m; $m = 200$ g $= 0.2$ kg; $KE = 1.6$ mJ $= 16\times10^{-4}$ J

Let v and x denote the instantaneous speed and displacement respectively of the bob. Obviously, $K = \frac{1}{2}mv^2$

$$16 \times 10^{-4} = \frac{1}{2} \times 0.2 \times v^2$$

$$\therefore \quad v^2 = 16 \times 10^{-3} (\text{ms}^{-1})^2$$

Also $v^2 = \omega^2\left[A^2 - x^2\right]$; therefore

$$16 \times 10^{-3} = \left(\frac{2\pi}{2}\right)^2\left[A^2 - x^2\right] \quad \left[\because \omega = \frac{2\pi}{T}\right]$$

$$= 10\left[A^2 - x^2\right] \qquad [\pi^2 \simeq 10]$$

$$16 \times 10^{-4} = \left(5 \times 10^{-2}\right)^2 - x^2$$

$$\text{or } x^2 = 9 \times 10^{-4}$$

$$\therefore x = \pm 3 \times 10^{-2} \text{m} = \pm 3 \text{ cm}$$

A.8.2.4 (C)

Using the relation

$v = \omega\sqrt{A^2 - x^2}$, we have

$$\alpha = \omega\sqrt{A^2 - \beta^2} \qquad (8.2.2)$$

$$\text{and}\left(\frac{\alpha}{2}\right) = \omega\sqrt{A^2 - \left(\frac{5\beta}{4}\right)^2} \qquad (8.2.3)$$

From Eqs(8.2.2) and (8.2.3).

$$\left[\frac{\alpha}{\frac{\alpha}{2}}\right]^2 = \frac{A^2-\beta^2}{\left(A^2-\frac{25}{16}\beta^2\right)}$$

$$\text{or } 4A^2 = 7\beta^2 \quad \therefore A = \frac{\sqrt{7}}{2}\beta$$

Substituting value of A in Eq(8.2.2), we have

$$\omega = \sqrt{\frac{4}{3}}\left(\frac{\alpha}{\beta}\right)$$

$$\text{and } T = \sqrt{3}\pi\left(\frac{\beta}{\alpha}\right) \quad \left[\because T = \frac{2\pi}{\omega}\right]$$

A.8.2.5 (B)

In SHM.; $a = -\omega^2 x$ i.e. a versus x graph is a straight line having a slope $-\omega^2$. Let ω_1 and ω_2 be the angular frequency of SHM, I and II respectively. From given graph

$$-\omega_1^2 = \tan 120° = -\cot 30° = -\sqrt{3} \text{ or } \omega_1^2 = \sqrt{3}$$

$$\text{And } -\omega_2^2 = \tan 150° = -\cot 60° = -\frac{1}{\sqrt{3}} \text{ or } \omega_2^2 = \frac{1}{\sqrt{3}}$$

$$\frac{T_1}{T_2} = \frac{\omega_2}{\omega_1} = \frac{1}{\sqrt{3}}$$

A.8.2.6 (A)

Given $x = \pm\frac{A}{\sqrt{2}}$. The instantaneous potential energy U is

$$U = \frac{1}{2}kx^2 = \frac{1}{4}kA^2$$

E = The total energy of SHM = $U_{max} = \frac{1}{2}kA^2$

K = The instantaneous kinetic energy = $E - U = \frac{1}{4}kA^2$

$\therefore \frac{U}{K} = 1$ and $\frac{E}{U} = 2$

A.8.2.7 (B)

The average kinetic energy, over one cycle of changes, is half the maximum kinetic energy (=E) of SHM. Therefore

$$E = K_{max} = 2\,K_{av} = 2\times16\text{ mJ} = 32\times10^{-3}\text{ J}$$

Given, $k = 10^{-3}\,\frac{\text{N}}{\text{cm}} = 10^{-1}\,\text{Nm}^{-1}$ we have

We know

$$\text{E} = U_{max} = \frac{1}{2}kA^2$$

$\therefore 32 \times 10^{-3} = \frac{1}{2} \times 10^{-1}A^2$ or $A = 0.8$ m

Also, $E = \frac{1}{2}mA^2\omega^2$, therefore $\omega = 2$ rad ω^{-1}

A.8.2.8 (A)

In Fig.Q.8.2.2(a) the potential energy in mean position = U_0 = 0.1 J. For x = 1 cm = 10^{-2} m; the instantaneous potential energy = U = 0.2 J. We know

$$U - U_0 = \frac{1}{2} mA^2\omega^2$$

$$U - U_0 = \frac{1}{2} k_1 x^2$$

$\therefore$ (0.2 – 0.1) = $\frac{1}{2} k_1 \left(1 \times 10^{-2}\right)^2$ or k_1 = 2000 Nm^{-1}

In Fig.Q.8.2.2(b); the potential energy (U_0) at the mean position, x = 0. Also U = 0.1 J for x = 10^{-2} m

$\therefore 0.1 = \frac{1}{2} k_2 \left(1 \times 10^{-2}\right)^2$ or k_2 = 2000 Nm^{-1}

$$\therefore \frac{k_1}{k_2} = 1$$

A.8.2.9 (C)

In general, the instantaneous potential energy U and instantaneous displacement x; in SHM. satisfy the relation

$$U = U_0 + \frac{1}{2} kx^2 \qquad (8.2.4)$$

where U_0 is potential energy in the mean position.

but we have $U = \alpha + \beta x^2$

Given relation is the same as Eq(8.2.4); therefore the motion of the particle is SHM. Also k = 2β. The time period, T; of motion is $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2\beta}}$

A.8.2.10 (B.)

Refer to Fig.A.8.2.1

ON = OA = l, OM= l

co θ

MN = ON – OM = $l(1-\text{co}\quad\theta)$

$PE = mgl(1-\text{co}\quad\theta)$

$KE = \frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2 = \frac{1}{2}ml^2\left(\frac{d\theta}{dt}\right)^2$

So, $\frac{1}{2}ml^2\left(\frac{d\theta}{dt}\right)^2 + mgl(1 - \text{co}\quad\theta) =$ a constant

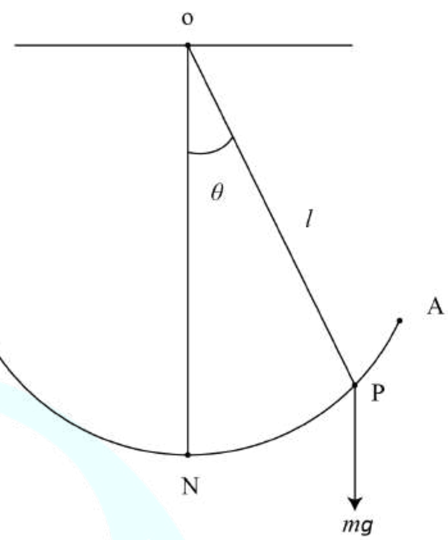


Fig.A.8.2.1

Dividing both sides by ml, we get

$\frac{1}{2}l\left(\frac{d\theta}{dt}\right)^2 + g(1 - \text{co}\quad\theta) = a$ constant

$\therefore\ \frac{1}{2}l\left(\frac{d\theta}{dt}\right)^2 - g\text{co}\quad\theta = a$ constant

A.8.3.1 (B)

$$T_1 = 2\pi\sqrt{\frac{M}{k_1}} \tag{8.3.1}$$

Given, $M' = M - \frac{M}{4} = \frac{3M}{4}$; $T_2 = T_2 + \frac{T_1}{2} = \frac{3T_1}{2}$;

$$\therefore T_2 = 2\pi\sqrt{\frac{M'}{k_2}} = 2\pi\sqrt{\frac{3M}{4k_2}} \tag{8.3.2}$$

From Eqs(8.3.1) and (8.3.2)

$$\left(\frac{T_1}{T_2}\right)^2 = \frac{\frac{M}{k_1}}{\frac{3M}{4k_2}} \text{ or } \left(\frac{2}{3}\right)^2 = \frac{4k_2}{3k_1}$$

$$\therefore \frac{k_2}{k_1} = \frac{1}{3}$$

A.8.3.2 (C)

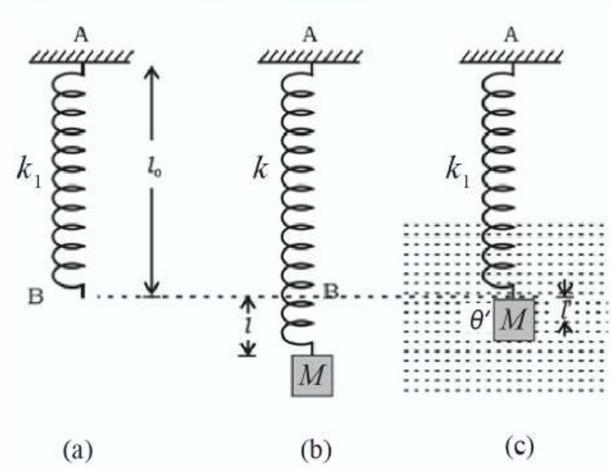


Fig.A.8.3.1

Fig.A.8.3.1(b) equilibrium position O of the system in air. OB = l , such that $Mg = kl$. When mass is immersed completely in liquid, due to buoyant force, $g_{eff} = g\left(1 - \frac{\sigma}{\rho}\right)$; σ and ρ denote density of liquid and material of mass M respectively.

Let l' be an extension in equilibrium position O' as shown in Fig.A.8.3.1(c). Obviously, $Mg' = kl'$

Since $g' < g$; $l' < l$. This means equilibrium position moves vertically upwards.

The time period (T) of oscillations is $T = 2\pi\sqrt{\frac{M}{k}}$

When mass is immersed inside liquid, neither M nor k changes; therefore the time-period of oscillations does not change.

A.8.3.3 (B)

At $t = 0$, the time period T_0 is

$$T_0 = 2\pi\sqrt{\frac{M_0}{k}}$$

where $M_0 = m_1 + m_2$

As liquid drips out, mass attached to spring decreases. Let M be the instantaneous mass of the arrangement. Then

$$M = M_0 - \alpha t$$

The instantaneous time-period, T, of oscillations is

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M_0 - \alpha t}{k}}$$

$$\text{or,} T^2 = \frac{4\pi^2}{k}\left[M_0 - \alpha t\right] \tag{8.3.3}$$

Eq(8.3.3) is the equation of the straight line between T^2 and t. At $t = \tau$; where $\tau = \frac{m_2}{\alpha}$; the entire liquid has dripped out. The mass attached to the spring now has a constant value m_1. The time period of oscillations is constant for $t > \tau$.

A.8.3.4 (C)

Given,

$$T_1 = 2\pi\sqrt{\frac{M}{k_1}} \tag{8.3.4}$$

$$T_2 = 2\pi\sqrt{\frac{M}{k_2}} \tag{8.3.5}$$

For, M is attached to S_1 and S_2 as shown; the springs are joined in parallel. The equivalent spring constant of the arrangement, k, is

$k = k_1 + k_2$

$$\therefore \quad T = 2\pi\sqrt{\frac{M}{k_1+k_2}} \tag{8.3.6}$$

Squaring and rearranging Eqs(8.3.4), (8.3.5) and (8.3.6) we have

$$\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$$

A.8.3.5 (C)

Springs of constant k and $3k$ are in parallel with one another. The equivalent spring constant $k' = k + 3k = 4k$.

k' is joined with a series with spring of spring constant $2k$. The equivalent spring constant, k_{eq} ; of the arrangement is

$$\frac{1}{k_{eq}} = \frac{1}{k'} + \frac{1}{2k} = \frac{1}{4k} + \frac{1}{2k}$$

$\therefore\ k_{eq} = \frac{4k}{3}$

The time-period, T; of oscillations of mass M is

$$T = 2\pi\sqrt{\frac{M}{k_{eq}}}$$

$$= 2\pi\sqrt{\frac{3M}{4k}} = \pi\sqrt{\frac{3M}{k}}$$

A.8.3.6 (A)

The mass M of the cylinder is $M = \pi R^2 L\rho$

Substituting value of M in given expression for T; we have

$$T = 2\pi\sqrt{\frac{\pi R^2 L\rho}{\pi R^2 \sigma g}} = 2\pi\sqrt{\frac{L\rho}{\sigma g}}$$

T is independent of R.

A.8.3.7 (C)

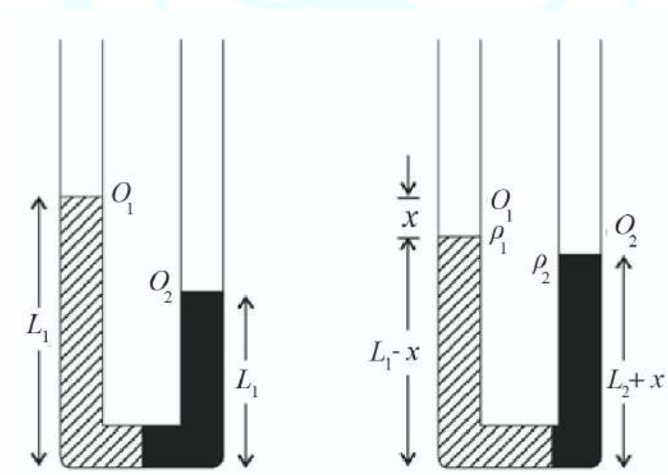


Fig.A.8.3.2

Fig.A.8.3.2(a) shows the equilibrium position of the arrangement. Obviously,

$$L_1\rho_1 g = L_2\rho_2 g$$

$$\text{or } L_1\rho_1 = L_2\rho_2 \tag{8.3.7}$$

Fig.A.8.3.2((b) shows the level of the liquids in the two limbs. For small, instantaneous displacement x, the net unbalanced restoring force, F, on the liquid column is

$$F = a[(L_2 + x)\rho_2 g - (L_1 - x)\rho_1 g]$$

$$= a(\rho_1 + \rho_2)gx \tag{8.3.8}$$

Let M be the total mass of liquid in the U-tube. Obviously

$$M = a[L_1\rho_1 + L_2\rho_2] \tag{8.3.9}$$

The equation of motion of liquid column is

$$M\frac{d^2x}{dt^2} = -[a(\rho_1 + \rho_2)g]x$$

$$\text{or } \frac{d^2x}{dt^2} = -\omega^2 x$$

where $\omega^2 = \frac{a(\rho_1+\rho_2)g}{M} = \frac{a(\rho_1+\rho_2)g}{a(L_1\rho_1+L_2\rho_2)}$

$$\frac{4\pi^2}{T^2} = \frac{a(\rho_1+\rho_2)g}{a(L_1\rho_1+L_2\rho_2)}$$

T = Time period of oscillations of the liquid column

$$T = 2\pi\sqrt{\frac{L_1\rho_2 + L_2\rho_1}{(\rho_1 + \rho_2)g}}$$

A.8.3.8 (C)

M = mass of liquid in U-tube = $2La\rho$. Substituting value of M in given expression for T; we have

$$T = 2\pi\sqrt{\frac{2La\rho}{2a\rho g}} = 2\pi\sqrt{\frac{L}{g}}$$

Note: T does not depend on (i) a and (ii) ρ. Therefore

$T_1 = T$ or $\frac{T_1}{T} = 1$

A.8.3.9 (C)

Time period, T, of a second pendulum is 2 s Therefore

$$T = 2\ s = 2\pi\sqrt{\frac{l}{g}}$$

(symbols have their usual meaning)

Inside lift; the time-period, T_1; of the pendulum = $T + \frac{T}{2} = 3$ s. Let g_{eff} be the effective value of acceleration due to gravity inside the lift. Them

$$T_1 = 3s = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

$\therefore \frac{4}{9} = \frac{g_{eff}}{g}$ or $g_{eff} = \frac{4g}{9}$

Since $g_{eff} < g;$ lift accelerates downward; say, with an acceleration a. We know

$$g_{eff} = g - a$$

$$\frac{4g}{9} = g - a$$

$$\therefore a = \frac{5g}{9}$$

A.8.3.10 (B)

The time period, T of a second pendulum having length l, is 2 s. Therefore

$$T = 2\text{ s} = 2\pi\sqrt{\frac{l}{g}} \tag{8.3.10}$$

When bob is immersed completely in liquid; because to the buoyant force due to the liquid; the effective value of g is

$$g_{eff} = g\left(1 - \frac{\sigma}{\rho}\right) = g\left(1 - \frac{1}{2}\right) = \frac{g}{2}$$

The time-period; T', of oscillations is

$$T' = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{2l}{g}} = \sqrt{2}.T = 2\sqrt{2}\text{ s}$$

A.8.3.11 (D)

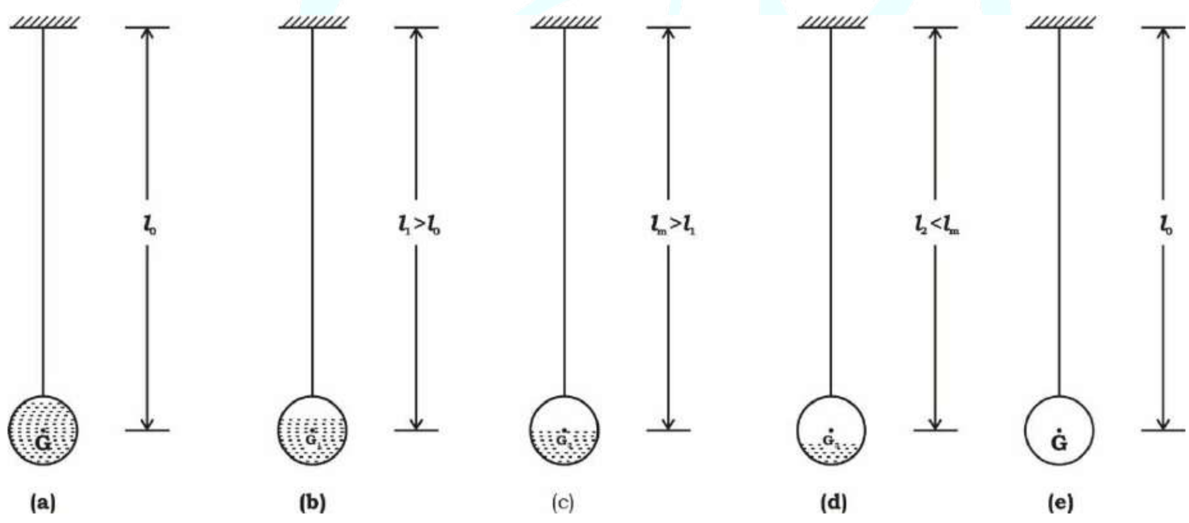


Fig.A.8.3.3

One is tempted to 'think' that mass of bob decreases (changes) as liquid drips out and time period should remain same, as time period DOES NOT depend on mass of bob. However, there is a change in length of pendulum (distance between point of suspension and CG of the bob) as liquid drops, as shown in Fig.8.3.3 The CG is at the lowest position when shell is HALF-empty as shown in Fig.A.8.3.3(c). From Fig A.8.3.3; it is clear that the length of the pendulum first increases, acquires a maximum value, l_m; and then decreases. Finally (when the shell is completely empty) the time period is the same as at the beginning. Therefore the time period first increases, acquires a maximum value, then decreases and finally is the same as at $t = 0$.

A.8.4.1 (B)

In the said equation $2b\frac{dx}{dt}$ gets added with an acceleration term. So, $2b\frac{dx}{dt}$ must have the unit of acceleration. So, the unit of b has to be that of (acceleration/velocity), i.e., s^{-2}/s^{-1}, hence, option B. The distractors have been framed keeping in mind that acceleration may be confused with force, particularly in case of option(D).

Note: It is crucial to understand that the dimension of b is the same as that of reciprocal of time. The term e^{-bt} appears in the solution of the differential equation of damped harmonic motion. Anything appearing as a power has to be dimensionless. More importantly, e^{x} is a power series in x. So, it can be understood that if x has a dimension, a havoc will be created and the series will become untenable. So, bt is dimensionless, and it can only happen when b has the dimension of reciprocal of time.

A.8.4.2 (B)

The highest order of derivative appearing in the equation is two. So, order is two. Again if x_1 and x_2 are two independent solutions, then $(m_1x_1 + m_2x_2)$ is also a solution, where m_1 and m_2 are scalars. So, the equation is linear, and thus, the degree is one. The distractors have been framed by keeping in mind the possible doubts one may have about any possible link between order of the derivatives, and their degrees.

Note: This question has been set despite fear of non-acceptance from the syllabus-oriented persons. Further, it may appear to be more skewed towards mathematics than physics. But the point to be noted is that there are significant links with physics, which are as under –

- In order to solve this equation, we need to have two boundary conditions, one in terms of velocity and the other in terms of displacement; in other words, the two orders of the differential equation are linked directly with the concept of phase, which is the state of motion governed by simultaneous consideration of velocity and displacement.

● The issue of the degree being one is the fountain-head of the 'principle of superposition', which finds application in so many phenomena associated with 'Oscillations and Waves'.

Most importantly, this material is meant for the teachers. So, some aberration has been ventured.

A.8.4.3 (C)

$\omega = \sqrt{\frac{k}{m}},\ \omega_1 = \sqrt{\omega^2 - b^2}$

So, $\omega = \sqrt{\frac{80}{0.28}} = 16.903;\ \omega_1 = \sqrt{285.783 - 0.0625} = \sqrt{285.6512} = 16.901$

So, the required error = $\frac{16.901 - 16.903}{16.901} \times 100\ \% = \frac{-0,2}{16.901} \% = -0.012\ \%$

One of the possibilities of getting distracted is connected with the sign of the error. Rest is about the calculation.

A.8.4.4 (A)

$A = A_0 e^{-bt};\ A = \frac{A_0}{2};\ e^{-bt} = \frac{1}{2}$

So, $ln\left(\frac{1}{2}\right) =- ln\ 2 =- bt$

So, $t = \frac{ln2}{b} = \frac{0.693}{0.25} = 2.773$ s

Improper handling of the logarithmic relation and wrong calculation are the possible reasons for distraction.

A.8.4.5 (B)

With symbols having their usual meanings, we have

$E = \frac{1}{2} kx^2$, where $x = x_0 e^{-bt};\ x^2 = x_0^2 e^{-2bt}$

So, $E = E_0 e^{-2bt}$

So, following the same argument as in case of A.8.4.4, we get that the

Required time = $\frac{ln2}{2b} = \left(\frac{1}{2}\right)\left(\frac{ln2}{b}\right) = \frac{1}{2}(2.773) = 1.387$ s

Incorrect determination of the power of e As a result of variation of energy with time, wrong handling of logarithmic formulas and miscalculation are the possible reasons behind distractions.

A.8.4.6 (C)

Option numbers (A), (B), (D) are interchangeably used to mean dangerous.

Option(C) has become synonymous with 'resonant' after several accidents were caused due to resonant vibrations, like the one in the Tacoma Narrows Bridge on November 7, 1940.

It is a situation where the affected persons are caught unaware about the sudden damage caused by resonant vibration, and thus the term was coined. Such accidents are testimony to the fact that Forced Vibration generally happens with very small amplitude, but it becomes significantly large in case of Resonance.

A.8.4.7 (A)

During a performance inside an auditorium, multiple reflections are caused primarily due to the walls and surfaces of other objects such as furniture, and even people; and the persistence of such reflected sound with its gradual dying away constitutes the phenomenon of Reverberation. Resonance has hardly any role here.

While in each of the cases of Options (B), (C), (D), we have a system which is subject to vibration, be it a suspension bridge (the only connectivity available or made temporarily available across violently flowing streams in difficult terrains accessed by soldiers under extremely compelling compelling circumstances), or a structure which has its foundation in the soil through which seismic waves (in cases with very small amplitude due to its source being far off) can travel, or an aircraft wing. Each has a natural frequency of vibration based on its mechanical properties, and there is a possibility of each being forced into vibration by some external cause. If the forcing frequency matches the natural frequency of vibration, then resonant vibration with very large amplitude will occur which can cause significant damage to the construction.

A.8.4.8 (A) For a lowly damped oscillator resonant frequency(p_r) is nearly equal to the natural frequency(ω) of an oscillator.

A.8.4.9 (D)

The equation of motion of a forced harmonic oscillator is given as

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega^2 x = f_0 \sin\ pt$$

Hence, the amplitude of forced oscillation depends upon b,ω and p.

A.8.4.10 (A)

We have

$$x(t) = A\ \sin(\omega_1 t + \phi_0)$$

given $\phi = -45°$

hence $\phi_0 = -45°, -1 = \frac{2bp}{\omega^2 - p^2}$

So, $2bp = p^2 - \omega^2$; $p^2 - 2bp - \omega^2 = 0$

$$\therefore p = \frac{2b + \sqrt{4b^2 + 4\omega^2}}{2} = b \pm \sqrt{\omega^2 + b^2}.$$

But the negative sign in the numerator is inadmissible. So, $p = b + \sqrt{(\omega^2 + b^2)}$. Attention needs to be paid on the possible errors that might be committed by way of not getting the phase angle correctly and incorrect determination of the value of tan(−45°).

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9.Waves

Module 9.1 Propagation of Waves

Q.9.1.1 Given below are the names of different kinds of waves.

(P) Light wave (Q) Sound wave (R) X-rays (S) Radio wave. Which among the following is/are of transverse nature?

Options

(A) (P) only

(B) (Q) and (R)

(C) (P), (R) and (S)

(D) (P)) and (R)

Q.9.1.2 It is said that mechanical waves require the inertia and the elasticity of the medium for its propagation. Now, the velocity of a progressive wave in a gaseous medium is given by $v = \sqrt{\frac{\gamma P}{\rho}}$ where P is the ambient pressure, ϱ = density of the medium and γ is the ratio of the specific heats of the gas at constant pressure and constant volume. Examine the statements given below-

(P) Only P represents elasticity of the medium

(Q) γ and P represent the inertia of the medium

(R) Only ρ represents the inertia of the medium

(S) Only γ represents the elasticity of the medium

Options:

(A) (P) and (Q) are correct

(B) (R) and (S) are correct

(C) Only (R) is correct

(D) (P) and (R) are correct

Q.9.1.3 (P), (Q), (R), (S) are four statements about the vibration of a tuning fork.

(P) The prongs execute transverse vibration

(Q) The prongs execute longitudinal vibration

(R) The stem executes longitudinal vibration

(S) The stem executes transverse vibration

Which among the above are correct?

- **(A)** P and Q
- **(B)** P and R
- **(C)** P and S
- **(D)** Q and S

Q.9.1.4 The velocity of propagation of a progressive wave is v if the amplitude of the wave is doubled then, the velocity of the wave

- **(A)** will become 2θ
- **(B)** will not change
- **(C)** will become 4θ
- **(D)** will become $\theta/2$

Q.9.1.5 In the path of a progressive wave two points are 14 cm apart. What is the phase difference between these two points if the wavelength is 28 cm?

- **(A)** $\frac{\pi}{2}$
- **(B)** π
- **(C)** 2π
- **(D)** $\frac{\pi}{4}$

Q.9.1.6 The source of a progressive wave in a non-dissipative medium is linear. What is the geometry of the wave fronts?

- **(A)** Spherical

(B) Planes

(C) Cylindrical

(D) Parabolic

Q.9.1.7 In the case of a plane progressive wave, if the maximum possible particle velocity is twice the wave velocity, then the magnitude of the amplitude of the wave in terms of its wavelength λ is

(A) $\frac{\lambda}{2\pi}$

(B) $\frac{2\lambda}{\pi}$

(C) 2λ

(D) $\frac{\lambda}{\pi}$

Q.9.1.8 The phase of vibration of a simple pendulum

(A) depends on its angular displacement only

(B) depends on its angular velocity only

(C) depends on both the angular velocity and the displacement

(D) depends on neither the angular velocity nor the angular displacement.

Q.9.1.9 a_1 and a_2 are the amplitudes of two progressive waves. If they superimpose, the maximum possible value of the resulting amplitude is

(A) $\sqrt{a_1^2 + a_2^2}$

(B) $\sqrt{a_1 a_2}$

(C) $a_1 + a_2$

(D) $\frac{a_1 a_2}{a_1 + a_2}$

Q.9.1.10 What is the dimension of k in the equation of a plane progressive wave given by.

$$y = a \sin k(vt - x)$$

(A) L

(B) L^{-1}

(C) T

(D) T^{-1}

Module 9.2 Stationary Waves

Q.9.2.1 Which one among the following statements is correct with respect to the reflection of waves?

(P) When reflection takes place from a denser to a rarer medium, the longitudinal waves are reflected in the same phase, but move in opposite directions.

(Q) When reflection takes place from a denser to a rarer medium, the longitudinal waves are reflected in the opposite phase and move in opposite directions.

(R) On reflection from a rarer to a denser medium, a longitudinal wave is reflected back with a change in phase.

(S) On reflection from a rarer to a denser medium, a longitudinal wave is reflected back with no change in phase.

Options:

- **(A)** (P) and (Q)
- **(B)** (Q) and (R)
- **(C)** (P) and (R)
- **(D)** (Q) and (S)

Q.9.2.2 Given below are statements regarding transfer of energy in progressive and stationary waves.

(P) Continuous transfer of energy takes place from particle to particle in a progressive wave

(Q) Transfer of energy takes place from a particle to another particle provided they are in same phase during the propagation of a progressive wave.

(R) In a stationary wave no transfer of energy takes place across the 'node' points.

(S) In a stationary wave no transfer of energy takes place across the 'antinode' points.

Examine the above statements and identify, which among them are correct.

Options:

- **(A)** (P) and (Q)
- **(B)** (P) and (R)
- **(C)** (P) and (S)

(D) (R) and (S)

Q.9.2.3 Identify the correct statement among the following for a stationary wave.

(A) No change of density takes place at the nodes

(B) Maximum change of density takes place at the antinodes.

(C) Velocity of particles is zero at antinodes.

(D) Velocity of particles is maximum at antinodes.

Q.9.2.4 The equation of a stationary wave is given by,

$y = 2 \cos 4x \sin 20t$

where x, y are in cm and t is in seconds.

Find the value of the velocity of the wave in cm s^{-1}

(A) 2

(B) 4

(C) 5

(D) 10

Q.9.2.5 The equation of a plane progressive harmonic wave is $y_1 = 2 \sin (4\pi t - \pi x)$, where x, y are in cm and t is in s. This wave reflects at a boundary and the reflected wave interferes with the original wave to form a stationary wave. Find the equation of the resulting stationary wave.

(A) $y = 4 \sin \pi t \cos 4\pi x$

(B) $y = 4 \sin 4\pi t \cos \pi x$

(C) $y = 2 \sin 2\pi t \cos \pi x$

(D) $y = 2 \sin 4\pi t \cos \pi x$

Q.9.2.6 A string of length 120 cm fixed at its two ends is undergoing stationary vibration. Considering the fundamental to be the first harmonic, the positions (in cm) of maximum change in pressure in the third harmonic are:

(A) 0 , 40, 80, 120

(B) 20, 60, 100

(C) 0, 20, 60, 100, 120

(D) 40, 80

Q.9.2.7 The material of a string undergoing stationary vibration is changed so that the density increases eight-fold. The tension in the string is then doubled. The frequency of the fundamental becomes '*x*' times its former value. Then *x* is equal to

(A) $\frac{1}{2}$

(B) 2

(C) 4

(D) $\frac{1}{4}$

Q.9.2.8 The second harmonic of the stationary vibration in an organ pipe of length 68 cm and one end closed is in unison with the vibration of a tuning fork. If the velocity of sound is 340 ms^{-1}, what is the frequency of the tuning fork?

(A) 300 Hz

(B) 350 Hz

(C) 375 Hz

(D) 400 Hz

Q.9.2.9 In an experiment to determine the velocity of sound by using the resonance tube method, the first two resonances occur at the lengths 24 cm and 70 cm respectively. If the velocity of sound so determined is 345 ms^{-1} then the frequency of the tuning fork is

- **(A)** 288 Hz
- **(B)** 324 Hz
- **(C)** 375 Hz
- **(D)** 405 Hz

Q.9.2.10 A stationary wave having wavelength λ originates at the point O. A, B, C are three points in the medium in which the wave exists. At a given instant of time, the displacement at A is y. if AB = BC = $\frac{\lambda}{2}$, then the corresponding displacements at B and C are-

- **(A)** y, y
- **(B)** $-y, y$
- **(C)** $y, -y$
- **(D)** $-y, -y$

Module 9.3 Beats and Doppler Effect

Q.9.3.1 The phenomenon of transient beats can be observed

(P) When two stationary waves of very nearly equal frequencies superpose.

(Q) When two progressive waves of very nearly equal frequencies superpose.

(R) When a driving vibration forces its vibration on a driven oscillating system and the frequency of the driven system approaches, that of the driver.

Which of the above statements is/are correct?

(A) only (P)

(B) only (Q)

(C) (P) of (Q), but not (R)

(D) only (R)

Q.9.3.2 An open pipe of length l and another 1% longer than it, produces beats of period 0.202 second when sounded together. If the velocity of sound is 340 ms^{-1}, then the value of l in cm is equal to

(A) 17

(B) 34

(C) 50

(D) 100

Q.9.3.3 Two closed organ pipes of the same length l, when sounded respectively in their fundamental and the second harmonic respectively, produce four beats per second. Then the velocity of sound is

(A) $2l$

(B) $4l$

(C) $8l$

(D) $9l$

Q.9.3.4 Two sonometers when sounded together produce two beats per second. They are made of wire of the same material and their lengths are equal. The tension in the second sonometer is 44% more than the first. The frequency of the fundamental of the first sonometer is

(A) 5 Hz

(B) 10 Hz

(C) 20 Hz

(D) 40 Hz

Q.9.3.5 T_1, T_2, T_3, T_4 and T_5 are five tuning forks of very nearly equal frequencies. Beats of periods 1 s, 0.5 s, 0.33 s and 0.25 s are produced when the combination first-second, second-third, third-fourth and fourth-fifth respectively are sounded together. Find the beat frequency when the first and the fifth forks are sounded together

(A) 1 Hz

(B) 2 Hz

(C) 5 Hz

(D) 10 Hz

Q.9.3.6 Bus No. 123 leaves terminus A and travels along a straight road to point B and the time of travel is more than five hours. The bus maintains a uniform speed of 10 ms^{-1} and four such buses leave terminus A every hour. A person travels from B towards A with a uniform speed of 5 ms^{-1}. How many such buses (No. 123) will the person encounter from the opposite direction in an hour?

(A) 4

(B) 5

(C) 6

(D) 8

Q.9.3.7 Two cars A and B, each moving with a speed of 10 ms^{-1}, are approaching each other on a long straight two-way road. What will be the apparent frequency of the horn of car A, as heard by the driver of car

B, before the cars cross each other, if the original frequency of the horn is 500 Hz and the speed of sound in air is 320 ms^{-1}?

(A) $\frac{33}{31} \times 500$ Hz

(B) $\frac{33}{33} \times 500$ Hz

(C) $\frac{33}{32} \times 500$ Hz

(D) $\frac{32}{31} \times 500$ Hz

Q.9.3.8 Two cars A and B, each moving with a speed of 10 ms^{-1}, are approaching each other on a long straight two-way road. Then what will be the apparent frequency of the horn of car A as heard by the driver of car B, after the cars cross each other? The original frequency of the horn is 600 Hz and the speed of sound in air is 300 ms^{-1}?

(A) $\frac{17}{16} \times 600$ Hz

(B) $\frac{16}{17} \times 600$ Hz

(C) $\frac{33}{34} \times 600$ Hz

(D) $\frac{32}{33} \times 600$ Hz

Q.9.3.9 A source of musical sound is placed exactly midway between a detector and a reflecting wall. When the source starts moving towards the reflecting wall, the detector observes beats with time period T produced by the direct and the reflected sounds. Find the speed of the source if the velocity of sound is v and the original frequency of the musical sound is n. Assume that the velocity of the source is much less than the velocity of sound.

(A) $\frac{4v}{nT}$

(B) $\frac{2v}{nT}$

(C) $\frac{v}{nT}$

(D) $\frac{v}{2nT}$

Q.9.3.10 Find the apparent wavelength of a line of wavelength 6000 Å, as observed from the light received from a distant receding source. It is given that the velocity of light is $3 \times 10^8 \, ms^{-1}$ and the velocity of the star along the line of sight is $6 \times 10^6 \, ms^{-1}$.

(A) 5800 Å

(B) 5880 Å

(C) 6050 Å

(D) 6120 Å

ANSWERS

Module 9.1 Propagation of Waves

A.9.1.1 (C)

Light waves, X-ray and radio waves are electromagnetic in nature with an electric field and a magnetic field oscillating in two mutually perpendicular planes and propagating along the third mutually perpendicular direction. So, light waves, X-ray and radio waves are transverse in nature. Hence, the (C). The sound wave is longitudinal in nature.

A.9.1.2 (D)

The bulk modulus = P, out of which P occurs as the bulk stress. So it represents elasticity. ρ, being the density, represents the inertia. Hence the option (D).

A.9.1.3 (B)

The prongs (when excited) either approach to or recede away from each other. Hence, the prongs execute transverse vibration. When they approach, the stem moves down, and when the prongs recede away from each other the stem moves up. So, the stem executes longitudinal vibration. Hence the option (B).

A.9.1.4 (B)

The velocity of propagation of a progressive wave does not depend on the amplitude of vibration. Hence the option (B)

A.9.1.5 (B)

Phase diff = $\frac{2\,\pi x}{\lambda} = \frac{2\,\pi}{28} x\; 14 \;=\; \pi,$ Hence the option (B).

A.9.1.6 (C)

At a particular time, the points which are equidistant from the line are at the same phase. They all lie on a cylinder with the linear source as its axis. Hence the option (C).

A.9.1.7 (D)

We know, $y = a \sin \frac{2\pi}{\lambda}(vt - x)$

Particle velocity = $\frac{\partial y}{\partial t} = \frac{2\pi a v}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x)$

maximum particle velocity = $\frac{2\pi a v}{\lambda} = 2v$

$\therefore\ a = \frac{\lambda}{\pi}$ Hence, (D).

A.9.1.8 (C)

Just a look at the expression $y = a \sin \frac{2\pi}{\lambda}(vt - x)$ shows that the phase $(vt - x)$ depends on both the velocity and the position. Hence the option (C).

Like any other mechanical system, the differential equation of a simple pendulum is of second order. Its solution is obtained by two integrations. The first integration yields the velocity and the second yields the position. Both are required for the determination of phases. Hence phase depends on both velocity and position i.e., displacement. Hence the option (C).

The expression for phase is $(\omega t + \delta)$, and so it apparently seems that phase does not depend on either velocity and displacement. But it has to be noted that the physical meaning of ω appears through the combined analysis of displacement vs. time graph and velocity-time equation.

A.9.1.9 (C)

$$x_1 = a_1 \cos (\omega t + \delta)$$
$$x_2 = a_2 \cos \omega t$$
$$x = x_1 + x_2$$
$$= a_1 \cos \omega t \cos \delta - a_1 \sin \delta \sin \omega t + a_2 \cos \omega t$$
$$= A \cos (\omega t + \varepsilon),$$

where

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta}$$
$$\varepsilon = \tan^{-1}\left(\frac{a_1 \ in\, \delta}{a_1 \cos \delta + a_2}\right)$$

$$A_{max} = \sqrt{a_1^2 + a_2^2 + 2a_1a_2}, \quad \text{Where } \cos\delta = 1$$

$$\therefore A_{max} = a_1 + a_2$$

Hence, (C)

A.9.1.10 (B)

$$y = a \sin k(vt - x)$$

$(vt - x)$ Has the dimension of length. The argument of the sine function should be essentially dimensionless. So, k must have the dimension of reciprocal of length. Hence, the option(B).

Module 9.2 Stationary Waves

A.9.2.1 (C)

The reflected wave always travels in a direction opposite to the original wave. When reflection takes place from a rarer to a denser medium, there is a phase change of 180°, but when the reflection is from a denser to a rarer medium, there is no change in phase.

Hence (P) and (R) are correct, So (C) is the right option.

A.9.2.2 (B)

In a progressive wave continuous transfer of energy takes place from one particle to the next. Hence (P) is correct. In a stationary wave nodes and antinodes are permanent points respectively of minimum (zero) and maximum displacement. So there is no transfer of energy across the 'node' points. Hence (R) is also correct. So, (B) is the right option.

A.9.2.3 (D)

Nodes are permanent points of minimum displacement and so maximum change of density takes place there, and the velocity of particles is zero. No change of density takes at the antinodes, the permanent positions of maximum displacement. So velocity of particles is maximum there.

Hence, (D) is the right option.

A.9.2.4 (C)

The equation of a stationary wave is given by,

$y = 2a \cos kx \ \sin \omega t$

Comparing with the given equation

Here, $k = \frac{2\pi}{\lambda} = 4 \text{ cm}^{-1}$, $\quad \omega = \frac{2\pi v}{\lambda} = 20^{\quad -1}$

$$v = \frac{\omega}{k} = \frac{20^{\ -1}}{4 \text{ cm}^{-1}} = 5 \text{ cm}^{\quad -1}$$

So, (C) is the right option.

A.9.2.5 (B)

$y_1 = 2\sin\ (4\pi t - \pi x)$

$y_2 = 2\sin\ (4\pi t + \pi x)$

$y = y_1 + y_2 = 2.2\sin 4\pi t \cos \pi x$

$= 4\sin 4\pi t \cos \pi x$

Hence, (B) is the right option.

A.9.2.6 (A)

Third Harmonic

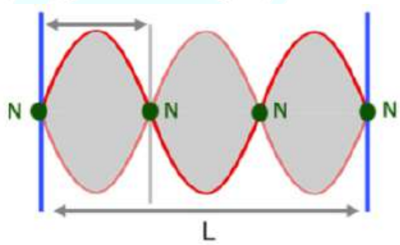

Fig. A.9.2.1

There will be three loops in the third harmonic (Fig. A.9.2.1), and so the segments will be $\frac{120}{3} = 40$ cm long. Therefore, nodes at 0, 40, 80, 120 cm. So, (A) is the correct option.

A.9.2.7 (A)

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{T}{\frac{\pi r^2 l\rho}{l}}}$$

$$\therefore\ n = \frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}}$$

$$x = \frac{n_2}{n_1} = \sqrt{\frac{T_2}{T_1}}.\sqrt{\frac{\rho_1}{\rho_2}} = \sqrt{2.\frac{1}{8}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

So, (A) is the correct option.

A.9.2.8 (C)

For the second harmonic, $\frac{3\lambda_2}{4} = l$

$$\therefore \lambda_2 = \frac{4l}{3}$$

$$\therefore \nu_2 = \frac{v}{\frac{4l}{3}} = \frac{3v}{4l} \text{ (where } v = \text{velocity of sound)}$$

$$\therefore \nu_2 = \frac{3}{4} \cdot \frac{340\times100}{68} = 375 \text{ Hz}$$

So, (C) is the right option.

A.9.2.9 (C)

For first resonance, $\nu_1 = \frac{v}{4l_1}$

For second resonance, $\nu_2 = \frac{3v}{4l_2}$

$$\therefore 2v = 4(l_2 - l_1)$$

Or,

$$v = 2\nu_2(l_2 - l_1)$$

Or,

$$\nu = \frac{v}{2(l_2 - l_1)} = \frac{345\times100}{2(70-24)} = 375 \text{ Hz}$$

Hence (C)is the right Option.

A.9.2.10 (B)

$$y = 2a \cos kx \sin \omega t$$

Displacement at A $\quad y = 2a\cos\frac{2\pi}{\lambda}x \sin \omega t$

Displacement at B, $= y\left(x + \frac{\lambda}{2}\right) = 2a\cos\frac{2\pi}{\lambda}\left(x + \frac{\lambda}{2}\right)\sin \omega t = 2a\, cos\left(\frac{2\pi x}{\lambda} + \pi\right) \sin \omega t$

$$\therefore y\left(x + \frac{\lambda}{2}\right) = 2a\cos\frac{2\pi x}{\lambda}\ \sin\ \omega t = -\ y$$

Displacement at C = $y\left(x + \frac{\lambda}{2} + \frac{\lambda}{2}\right) = y(x + \lambda)$

$$= 2a\cos\frac{2\pi}{\lambda}(x + \lambda)\sin\ \omega t$$

$$= 2a\cos\left(\frac{2\pi x}{\lambda} + 2\pi\right)\ \sin\ \omega t$$

$$= 2a\cos\frac{2\pi x}{\lambda}\sin\omega t = y$$

Hence (B) is correct.

Module 9.3 Beats and Doppler Effect

A 9.3.1 (D)

The crux of the matter is the word 'transient'. In all the given cases, beats will be produced. But beat frequency will be a variant for a brief moment of time when the forced system approaches the frequency of the driving system while executing forced vibration. Hence (D).

A. 9.3.2 (B)

$$\Delta\nu = \frac{v}{2}\left[\frac{1}{l} - \frac{1}{(1.01)l}\right] = \frac{v}{2l} \times \frac{.01}{1.01} = \frac{v}{2l} \times \frac{1}{101} = \frac{1}{0.202}$$

$$\therefore \frac{v}{2l} = \frac{101\times1000}{202} = 500$$

$$\therefore l = \frac{v}{1000} = 0.34 \text{ m} = 34 \text{ cm.}$$

Hence, **(B)** is correct.

A. 9.3.3 (C)

$$\frac{3v}{4l} - \frac{v}{4l} = 4$$

$$\therefore \frac{2v}{4l} = 4$$

$\therefore v = 8l$ Hence the option (C) is correct.

A.9.3.4 (B)

$$\frac{1}{2l}\sqrt{\frac{T}{m}}(1.2 - 1) = 2$$

$$\therefore \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{2}{0.2} = 10 \text{ Hz,}$$

Hence **(B)** is the correct option**.**

A. 9.3.5 (D)

$$\nu_1 - \nu_2 = \frac{1}{1} = 1 \text{ Hz}$$

$$\nu_2 - \nu_3 = \frac{1}{0.5} = 2 \text{ Hz}$$

$$\nu_3 - \nu_4 = \frac{1}{0.33} = 3 \text{ Hz}$$

$$\nu_4 - \nu_5 = \frac{1}{0.25} = 4 \text{ Hz}$$

$$\therefore \nu_1 - \nu_5 = (\nu_1 - \nu_2) + (\nu_2 - \nu_3) + (\nu_3 - \nu_4) + (\nu_4 - \nu_5)$$

$$= 1 + 2 + 3 + 4$$

$$= 10 \text{ Hz}$$

Hence (D) is the correct option.

A. 9.3.6 (D)

Number of buses crossing the person /hour $= n' = \left(\frac{v}{v-v_s}\right)n_0 = 4\left(\frac{10}{10-5}\right) = 4 \times 2 = 8$

Hence (D) is the correct option.

A. 9.3.7 (A)

$$n' = \frac{v+v_0}{v-v_s}n = \frac{320+10}{320-10}\times 500 = \frac{33}{31}\times 500 \text{ Hz.}$$

Hence (A) is the correct option.

A. 9.3.8 (B)

$$n' = \frac{v-v_0}{v+v_s}n = \frac{330-10}{330+10}\times 600 = \frac{32}{34}\times 600 = \frac{16}{17}\times 600 \text{ Hz.}$$

Hence (B) is the correct option.

A. 9.3.9 (D)

$$n_D = \frac{v}{v+v_s}, n_R = \frac{v}{v-v_s}$$

$$\therefore \frac{1}{T} = n_R - n_D \quad = \quad nv\left(\frac{1}{v-v_s} - \frac{1}{v+v_s}\right) \quad = nv \cdot \frac{2v_s}{v^2-v_s^2} = nv \cdot \frac{2v_s}{v^2} \; (\because v >> v_s)$$

$$\therefore \frac{1}{T} = \frac{2nv_s}{v} \text{ or } v_s = \frac{v}{2nT}$$

Hence, (D) is the correct option.

A. 9.3.10 (D)

$$v = \frac{\Delta\lambda}{\lambda}c, \; \therefore \Delta\lambda = \frac{\lambda v}{c} = 6000 \text{ Å} \times \frac{6\times10^6}{3\times10^8}$$

$\therefore$ The apparent wavelength $= 6120$ Å

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